Properties, Possibilities, and Ordinary Things

Toward the Pleasures of Platonism Without the Pain

!!developmental draft (v 2.0)!!

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April 29, 2018

Preface

As indicted by the subtitle, I suppose platonism is associated with certain pleasures and pains. Platonic entities play a role in philosophy from the analysis of fiction to mathematics. A pleasing proposal is the simple one that there are the fictional characters, numbers, and the like, that seem to be objects of such discussion. But if these are conceived as things outside of space and time, with which we have no causal contact, there is a certain pain admitting that they are. It is mysterious what is the nature of these entities, and how we know about them. Like the Easter Bunny, it is natural to sweep them away once we know what the world is like. The thesis of this book is that it is possible to have the pleasures of platonism without the pain. Platonic entities are rather more like the northern lights than the Easter Bunny. They have indeed a rare beauty, but remain explicable on the ground of experience with ordinary objects. I do not propose that there is an "easy" way into metaphysics. This explication requires work, but that is pleasure too.

The book forwards a project on which I have worked since my dissertation, some years ago now. Central papers are, "Worlds and Modality," "In Defense of Linguistic Ersatzism," and "Things and *De Re* Modality." I did not set out to explicate platonism as such. In the dissertation, under the direction of Michael Jubien, I took up questions about the nature of possibility and necessity. Starting as philosophers are wont to start, I considered especially "possible worlds," or ways the world could be, taken seriously as things. Since then, in the first two of the mentioned papers, I have allowed that there are coherent notions of abstract possible worlds, and that possible worlds have a certain utility. But such worlds are not the ultimate ground for modal truth, truth about possibility and necessity. Rather, possible worlds, and modal truths more generally, have their ground in the actual intrinsic structure of non-modal properties. "Things and *De Re* Modality" extends the picture to an account of (*de re*) modal properties of particular things, grounded also in the way actual things and properties are.

So far, perhaps, so good. But, depending as it does on abstract worlds and properties, the account of modality is hostage to the metaphysics of abstract worlds

and properties. In this work, I develop and extend the approach to grounding to see the entire abstract platonic menagerie, including abstract properties and possibilities, as grounded in the way things actually are. Such grounding is the way to the pleasure without pain. Naturally, I hope to clarify and improve what has gone before, and some of the main themes are anticipated or suggested in earlier work. But this book goes well-beyond anything from before. The result of extending the grounding thesis from possibility and necessity to abstracta more generally is a broad account of what there is, including accommodation both of pressures leading to nominalism and to platonism: On the one hand, abstract things are as real as rocks and trees, lumps and statues. On the other, abstract things have their ground in the actual concrete world. So there are abstract things, but they are no embarrassment, even to one whose starting point is just the particular concrete objects given in experience. Thus the project expands to engage the larger debate over nominalism and platonism, and to attempt a sort of reconciliation between opposing camps. The current work begins with pressures from the work on modality, and includes application back to that project. In the present context, however, these concerns frame the larger grounding proposal.

I have done what I can to keep the text interesting, and so to keep the reader engaged throughout. There are many cases and examples which put flesh on the bones of the overall structure for application to substantive philosophy. In so doing, however, I may give unnecessary grist for reactions against the view, and so motivations to commit the work "to the flames." I ask that, to whatever extent possible, the reader distinguish between cases and examples, and the larger theses they are meant to illustrate and defend. If the larger theses are established, this work is a success.

Special thanks to my colleagues Matthew Davidson, John Mumma and Darcy Otto for comments on different versions of the work. I have benefited by comments from Alfredo Watkins... Initial portions were presented in a seminar course at California State University, San Bernardino, with Davidson and Otto my co-instructors. Thanks also ...

T.R.

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Named Principles

Chapter 1

The End and the Beginning

Central to philosophical metaphysics is the question what sort of things there are, what are the fundamental constituents of reality. As with other theorizing, one aims to account for data — in this case, the nature of our experience, along with things we want to say about science, value, and the like — without postulating more than is required. And, as in other cases, there may be tensions between theoretical adequacy on the one hand, and plausibility or simplicity on the other. The debate over platonism and nominalism is a case in point.

Plato is famous for his doctrine that there are eternal, unchanging forms, distinct from anything in the physical world of experience. This is an explanatory hypothesis. Many have thought that the hypothesis, or some relative of it, has work yet to do. At the same time, the postulation of these things distinct from anything of the sort investigated by science may seem incredible — our grounds for thinking there are things do not extend so far beyond the physical world as to reach a realm of abstract forms. The aim of this work is, ultimately, to work toward a reconciliation of opposing pressures: I shall argue that it is possible to have the explanatory benefit of the platonic universe, where the elements of this universe are no less "ordinary" than the rocks, tables and chairs with which we interact every day.

The aim of this introductory chapter is to frame the issue. I begin in the first section by characterizing the distinction between abstract and concrete objects, and then contemporary platonism and nominalism as contrasting views about abstract objects. In the second section, I explore what I take to be fundamental reasons from philosophical method for the different positions. The reason there are both nominalists and platonists is that there are good reasons for both nominalism and platonism. Reasons for accepting platonism trade on difficulties for nominalism, but from these very reasons there is a problem about the primitives with which platonism begins.

Similarly, reasons for accepting nominalism trade on difficulties for platonism, but from these reasons there is a problem about the primitives with which nominalism begins. In the third section I develop these problems about primitives. In the last section, I sketch the overall plan of attack in the rest of this work, and so the way I hope to reconcile at once reasons for accepting both sorts of views, without succumbing to problems. Of course given history, this is a tall order. I will be happy to have made reasonable progress by the end!

1.1 Platonism and Nominalism

In their contemporary dress, platonism and nominalism are contrasting positions about the existence and nature of abstract objects. But it is no simple matter to say what abstract objects are supposed to be. In this section, I begin with a characterization of abstract objects and then, with this characterization in hand, turn to the substantive theses of platonism and nominalism that will be our main concern.

1.1.1 Abstract Objects

There is substantial intuitive agreement about which things count as abstract and which do not. Supposing there are such things, sets, properties, propositions, biological species, languages, word types, fictional plots and and fictional characters are abstract. In addition, an institution such as the Supreme Court, which is not itself identical with any particular physical structure or individuals may seem to be abstract. Again, supposing there are such things, rocks, protons, word tokens, leprechauns, God, immaterial minds, particular living things, and events such as the Iraq wars are not abstract. Though it may seem a strange usage for some of the cases, I will say things on the latter list are "concrete." There should also be room on the latter list for donkeys in Lewis's plurality of spatiotemporally and causally isolated worlds, along with the worlds themselves. Despite agreement about cases, it has proven difficult to produce a general characterization of the difference between objects on the different lists. But it seems important to have such a characterization. For we shall want to reason generally about such objects. And it would seem important to rest such reasoning on general features of objects under discussion.[1]

Toward this end, I begin by proposing a necessary condition, such that all abstract objects meet it. The condition is also sufficient, under the supposition of physicalism

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[1]Much contemporary discussion of the abstract/concrete distinction depends on Lewis, *On The Plurality of Worlds*, 81-86. Burgess and Rosen, *A Subject With No Object*, 13-25 along with Rosen, "Abstract Objects" are helpful summaries of options and issues, the latter with further references.

for concreta — with God, immaterial minds, and the like set to the side. With the necessary condition in-hand, I gesture toward a generalization that is also sufficient. The account will help us to see how standard proposals go in the right direction, without being entirely adequate.

Necessary Conditions.   On a traditional account, abstract objects lack some range of properties associated with objects that are concrete. A natural first thought is that concrete objects, or at least the ones we may be inclined to take seriously, are ones that are located in space in time, and participate in causal interactions. Thus, an object is *abstract* iff it is not one of these — iff it is not located in space, or not located in time, or does not participate in causal interactions. But it is not obvious that all abstract objects meet these conditions. Burgess and Rosen suggest a nice fantasy according to which at a distant time in a galaxy far far away is a perfect duplicate of our planet. Thus there is on this planet a duplicate of our Supreme Court. Though one may be inclined to say that there is in the universe a single abstract number one, it seems that there are distinct Supreme Courts, one there and then, and another here and now. So the Courts have spatial and temporal locations. Further, on some natural accounts, languages, plots, and fictional characters are the sort of thing that are created, and are temporally located insofar as they begin and develop over time. Similarly, sets may seem to have the same spatial and temporal locations as their members. Amie Thomasson, *Fiction in Metaphysics*, makes a proposal along these lines for fictional characters, and Penelope Maddy, *Realism in Mathematics*, one for sets. If such accounts can be right, lack of spatial or temporal location are not necessary for a thing to count as abstract. Of course, one may dig in and simply deny Maddy's proposal about the location of sets. And one may adopt a picture on which languages, plots, and fictional characters are not so much "created" as Thomasson would have them, as "selected" from a range of abstract things, themselves without location. But it seems question-begging to build the location conditions for abstracta into their very characterization.

The causal condition is more difficult to evaluate. On the one hand, languages, fictional characters, courts, and the like seem to be the sorts of things that are created and so caused. And there is some sense in which they have effects — it is important that, say, the Supreme Court has nine justices instead of one. But consider a modified version of a case from Rosen: Say John is thinking of the Pythagorean Theorem and falls into a well. His falling into the well because he is thinking of the Theorem may seem parallel to his falling into the well because he trips over a rake — and to whatever extent they are parallel, the Theorem and the rake seem to take on causally parallel roles. Still, it is natural to think that there is some difference between the

way the rake and the theorem matter. Short of a full-blown analysis of causation, which I cannot give, it will be difficult to separate these cases. And even with such an account, it will be difficult to separate them uncontroversially. If there is such a difference, though, there may be room to say that abstracta do not participate in causal interactions as do concreta, and to recast the causal proposal along these lines. I try to advance this point a bit below.

So these proposals are not entirely satisfactory. I do think there is something correct about them, though there is work to say what it is. We can make progress by generalizing the account of what abstracta are not, and so of what concreta are (and here is where our preliminary supposition of physicalism for concreta comes to play). Start by considering Lewis's doctrine of Humean supervenience.

> consider signifi-cance of quantum mechanics

> All there is to the world is a vast mosaic of local matters of particular fact, just one little thing and then another.... We have geometry: a system of external relations of spatiotemporal distance between points. Maybe points of spacetime itself, maybe point-sized bits of matter or aether or fields, maybe both. And at those points we have local qualities: perfectly natural intrinsic properties which need nothing bigger than a point at which to be instantiated. For short: we have an arrangement of qualities. And that is all. There is no difference without difference in the arrangement of qualities. All else supervenes on that. (Lewis, "Introduction," ix-x)

On this view, an arrangement of qualities is the base on which all else supervenes. It is not my intent to defend this doctrine! Humean supervenience is hardly uncontroversial. Much controversy surrounds the supervenience claim — whether resources are adequate to account for some phenomenon or another.[2] Still, Humean supervenience is a *nominalistic* proposal. It is reasonable to ask if abstracta are are among the objects that supervene on the base. And we make progress if it is possible to understand our platonism from the presumptively nominalistic foundation. For now, then, let us take it as a start.

Suppose, then, the distribution of qualities. Observe that we require no claim according to which qualities are, in Armstrong's phrase, "junior substances" capable

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[2]Thus given just a distribution of qualities at one instant and then another, it is not clear whether the view has resources to account for homogeneous rotating matter as such. And Lewis addresses a concern about objective chance. On the first concern, see Callender, "Humean Supervenience," Hawley, "Persistence and Non-Supervenient Relations," Zimmerman, "Temoral Parts and Supervenient Causation," Lewis, "Zimmerman and the Spinning Sphere," and Zimmerman, "Reply to Lewis." On the latter, see Lewis, "Introduction," "A Subjectivist's Guide to Objective Chance," "Postscripts to 'A Subjectivist's Guide to Objective Chance'," and "Humean Supervenience Debugged."

of independent existence, and no commitment to an account on which things are bundles of qualities — whatever this may come to (see Armstrong, *Universals*, 114-115). Rather, for the proposal that there is a distribution of qualities, it is enough that there are things with qualities at spacetime locations. Given this, there is a distribution of qualities. Also, I do not mean to engage in controversy over physics. Maybe, for whatever reasons, there are problems about point instantiation of properties and so about point qualities; and maybe we shall be required to revise our vision of qualities themselves in favor of quantum mechanical state descriptions. Still, in some sense, physics has to be telling us about the way ordinary things are at various spacetime locations. So let the account of ultimate reality be the true one. Then, subject to what we may think of as simplifying assumptions, this discussion begins where that one leaves off.

Say a thing has a quality as *constituent* just in case it is characterized by the quality or has it as a part. Then our basic idea is that a concrete object is one that has some quality or qualities as constituents. Thus a wide range of entities will count as concrete. Let us suppose that instances of mass, charge, color and the like count as qualities — I expect that these will do as well as any. Then a tree is concrete insofar as it is characterized by particular qualities of mass, charge and the like; as is an event like a parade which may be characterized by, say, various colors. And a particular quality will count as concrete insofar as it is part of itself. Then necessarily, an abstract object is not concrete.

AN If a thing is *abstract* then no particular qualities are constituents of it.

Observe that the current proposal is independent of the full-blown supervenience thesis. It is enough that rocks, protons, events and the like have some particular qualities as constituents and so come out as concrete.[3] One might object that no actual quality is a constituent of a donkey in one of Lewis's non-actual worlds. But the condition might be understood so as to apply to qualities in arbitrary worlds. Then the donkey does not come out abstract on our account. We can see better how all this works, by making connections to traditional proposals.

From AN, abstract objects lack particular qualities. So far, this is like the traditional approach on which abstract objects lack features in a certain range. But consistent with AN, abstract objects might lack features beyond particular qualities, and especially ones associated with the particularity of qualities. Insofar as every quality is located, lack of location would seem to be sufficient for a thing to meet

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[3]The status of spatiotemporal points and regions is unclear. If these count as things and as concrete, then include them among the particular qualities for the purposes of AN.

the condition from AN. Thus, if a thing lacks spatiotemporal location, no particular qualities are constituents of it. So if a thing lacks spatiotemporal location, it satisfies the condition necessary for being abstract. But lack of location is not required. A thing like the Supreme Court might be located, but lack particular justices and so particular qualities of them. So a thing might be located but abstract all the same. So our account has the potential to come apart from the spatiotemporal one, but in just the way we want.

Supposing it can be made out, the causal criterion may in fact track with the present proposal. It is natural to think that a thing has its causal properties because of its qualitative character. Thus a particular bit of table salt is soluble just because of its particular nature and structure. But if abstracta lack qualities as constituents, they would seem to lack precisely that which is the ground of ordinary causal behavior. So from our condition one might expect failure to participate in ordinary causal interactions as a mark of the abstract. As before, it is important to spell out just how such "ordinary" causal behavior differs from whatever interaction there may be with abstracta, if such there be. We shall have something to say about this in later chapters. For now, I simply observe that there is reason to think that we have pried underneath whatever there is to the causal criterion, in the direction of its underlying source.

Sufficient Conditions. Though no abstracta have particular qualities as constituents, neither do God or immaterial minds. But these things are not abstract. So AN is not a sufficient condition, or at least it is not apart from a supposition of physicalism for concreta. And similarly for the spatiotemporal and causal conditions. On some accounts at least, God is not located in space and time. And one might imagine a particularly reclusive, never incarnated, spirit without a spatiotemporal location, and never involved in causal interactions. Yet such a thing would not therefore be abstract.

But we can generalize our approach to reach conditions that are necessary and sufficient. The basic idea so far is simple: It may be possible to distinguish a notion of things or substances distinct from their qualities; and of qualities in terms of which the things have an analytic account. At any rate, given the qualities, arbitrary combinations of them are taken as sufficient for identification of the full range of concreta. And if a thing is not concrete, then it is abstract. The problem is that we do not start out with the full range of concrete substances with their corresponding fundamental qualities, but only with physical ones. If we were to start with a generally applicable notion of substance and the qualities of them, the corresponding range of concreta would be correspondingly generalized so that abstract objects would be just the ones that are not concrete.

One general account of substance is developed in terms of independence in Hoffman and Rosenkrantz, *Substance Among Other Categories*. Though an account of substance in terms of independence has deep historical roots, it is no simple matter to spell out the details, and their approach is too complex for me to develop here. Suffice it to say that on their account, necessary and dependent abstracta, along with events and the like, fail to qualify as substances. But, as Hoffman and Rosenkrantz tell the story, God, immaterial minds and material objects all qualify. It is not my purpose to defend or explicate this view. Let us simply suppose something along these lines is on offer. And suppose we have some account of *fundamental qualities* in terms of which the substances have an analytic account. For physical objects, these should be the same as before. If God is simple, the result may be a single quality. But we may remain neutral about that. Then things work as before. A concrete object is one that has as *constituents* some fundamental quality or qualities, in the sense either that it is characterized by the qualities, or has them as parts. Observe that the concrete objects include the substances but, as before, may include events or the like whose analysis includes their qualities. And, necessarily an abstract object is not concrete.[4]

  AS A thing is *abstract* iff no fundamental qualities are constituents of it.

Supposing the account of substances is such as to include God and immaterial minds, fundamental qualities include qualities of them, so they are not abstract. But, additionally, we are in a position to allow that the fundamental qualities are constituents of concrete things in addition to the substances so that spiritual or mental events, say, such as the last judgment, or the judgment that this account of abstract objects is insane, are concrete. And, at the same time, we do not collapse, for example, a divine Supreme Court, or set of divinities into concreta. So things seem to be working as they should.

Without a fully developed account of substances and their fundamental qualities, we do not have a fully developed analysis of abstracta. However we have at least a rough characterization of the full range under consideration. And for the following, I propose to set aside complications which arise from non-physical concreta, and simply to assume that all the fundamental qualities are the qualities from the spatiotemporal distribution. I do not mean to decide for or even favor physicalism against other views. We shall return to the question at the end. However, I think basic questions for platonism and nominalism remain the same. And issues are best framed without unnecessary complications. With this assumption about fundamental qualities, the

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[4]Hoffman and Rosenkrantz offer a rather different account of abstract objects than the one on offer here.

necessary and sufficient condition AS takes on the same clarity as the original necessary condition AN. And we are in a position to think substantively about abstract and concrete objects, or at least the range of them properly framed by our assumption.

1.1.2 Grounded Platonism

Platonism and nominalism are theses about the existence and nature of abstract objects. Say a thing is *independent* of all concrete objects when it might exist against the background of different configurations of concrete objects and even without concreta altogether. Roughly, a thing exists *objectively* just in case it is independent of mental states, in the sense that it might exist against the background of different configurations of mental states, and even without mental states altogether. But this fails to identify the objective existence of mental states. We do better with a restriction on the relevant mental states. Say a thing has a property under the *condition* of its analysis. Then a property is *subjective* iff its condition is dependent on the attitudes of a person, group, tradition or the like directed at (about) the satisfaction of that condition. A property is *objective* iff not subjective. So a skirt's BEING FASHIONABLE is subjective insofar as its condition depends on attitudes about length, color and cut characteristics; a ball's BEING ROUND is objective insofar as the condition being equal distant from the center is attitude independent. Similarly, where an object has some "conditions of existence," it exists subjectively just in case those conditions are dependent on attitudes directed at (about) their satisfaction, and objectively if not. So the attitudes by virtue of which a skirt is fashionable may themselves exist objectively.[5] If on the other hand it is as a result of language or some convention that conditions result in things, then the things exist subjectively rather than objectively.

With this said, we can distinguish different forms of platonism and nominalism. I shall distinguish *hard* and *soft* versions of platonism, and *hard* and *soft* versions of nominalism. On the soft version of platonism, there are objectively existing abstract objects. On the hard version of platonism, at least some abstract objects are independent of all concrete objects. The soft version of nominalism denies such independence: all objects are dependent on the concrete. On the hard version of nominalism, all objects are in fact concrete. Thus we have,

HP There are abstract objects, some independent of the concrete.

SP There are objectively existing abstract objects.

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[5]This exposes an important point: underlying subjective facts are objective ones according to which relevant attitudes obtain. One might say that any subjective object or property is part of a broader objective context including the object and whatever has the relevant attitudes.

SN All objects are grounded in, and so dependent upon, the concrete.

HN All objects are concrete.

The hard version of platonism is incompatible with either version of nominalism. And the hard version of nominalism is incompatible with either version of platonism. Perhaps one or both of these are the traditional versions of the views, and explain why platonism and nominalism are often thought to be incompatible. It remains to be seen whether arguments for platonism and nominalism support their soft or hard versions, and whether the soft versions somehow collapse into their hardened counterparts. However, SP and SN are not obviously inconsistent — for there might be objectively existing abstract objects grounded in ones that are concrete. Let the conjunction of SP and SN be *grounded platonism* (GP). This is the view that I shall defend.

I think we can clarify this view, and set up discussion of traditional strengths and weaknesses for platonism and nominalism, by keeping before us a difficulty about the supposed links between concrete and abstract things. Observe that SN may seem to be a version of Lewis's Humean supervenience thesis. According to Humean supervenience, all objects supervene on a distribution of concrete qualities. If this comes to a grounding or dependence relation, then SN. And if abstracta are among the things that supervene, then SP and so grounded platonism.

But supervenience theses are typically cashed in modal terms, and as a relation between properties. Here is approach designed to accommodate supervenience between classes of things. Say the Ps and Qs are things, and a *distribution* of Qs is simply the Qs as constrained by certain properties and relations.

ST The Ps *supervene* on the distribution of Qs iff in any world in which there are the Ps there is a distribution of Qs that necessitates the Ps.

A distribution of Qs necessitates the Ps just in case necessarily if there is the distribution of Qs, then there are the Ps — just in case any world with the distribution of Qs has the Ps.[6] Thus we might want to say that some complex things supervene upon their parts just in case for any world in which in which there are the things, there is a distribution of the parts such that necessarily the distribution entails the existence of the things. And, similarly, on the thesis of Humean supervenience, complex things supervene on the distribution of qualities — a given distribution of qualities entails a distribution of things.

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[6]This is a version of *global* supervenience. There is, of course an enormous literature. To start, see Kim, "Concepts of Supervenience," along with his "Supervenience for Multiple Domains."

Things are set up so that abstract objects are not concrete. So even though the abstract supervenes on the concrete, if there are abstract objects, HN fails.[7] And if the abstract supervenes on the concrete, there is no world where there are abstract objects apart from some distribution of ones that are concrete. Thus, if the abstract supervenes on the concrete, HP fails. And if the abstract supervenes on concrete objects which are themselves appropriately independent of the mental, a supervenience thesis makes abstract objects independent of the mental. So a supervenience thesis may justify SP. And grounded platonism, which adds SN to SP may thus seem closely related to a thesis of supervenience for abstracta.

But, involving as it does a notion of grounding, SN is not a mere supervenience thesis the sense of ST. As Kim emphasizes in "Supervenience as a Philosophical Concept," modal accounts along the lines of ST capture at best a notion of *covariance* and do not themselves develop notions of *dependence* or *grounding*. Given that the Ps and Qs covary across worlds, and so that something like ST is satisfied, it is left open what is the relation that explains their coming together. And without this further explanation, the covariance might appear itself as a fundamental philosophical mystery. Thus, after observing that moral non-naturalism requires a supervenient link between natural and non-natural moral properties, Mackie wants to know "*what in the world*" is the nature of this link. Thinking of Moore, "It is not even sufficient," he says, "to postulate a faculty which 'sees' the wrongness: something must be postulated which can see at once the natural features that constitute the cruelty, and the wrongness, and the mysterious consequential link between the two" (*Ethics*, 41). Kim suggests that it is the role of particular moral theories to supply just such a link. And similarly, Kim says, "if we want to promote the doctrine of psychophysical supervenience, intending it to include a claim of psychophysical dependence, we had better be prepared to produce an independent justification of the dependency claim which goes beyond the mere fact of covariance between mental and physical properties" (147-8). And he thinks this is the role of particular theories in philosophy of mind. And similarly for our case as well. Though we accept a supervenience thesis along the lines of ST for abstract objects, it sits on a grounding thesis along the lines of SN. And if we accept a supervenience thesis, we take on also the burden of explaining the grounding in this relatively robust sense.[8] The nature of this burden will come into greater focus as we turn now to preliminary motivations for the different positions.

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[7]So I do not to take on board Armstrong's doctrine of the "ontological free lunch" according to which "the supervenient is not something additional to what it supervenes upon." See, for example, Armstrong, "*A World of States of Affairs*," 12-13. As illustrated by this case, I find the doctrine fundamentally mysterious. There is not much nourishment in the supervenience of the Ps, if they are just the Qs and so already on the table. I think there is nourishment to be had, but we shall have to work for our meal.

[8]As motivation for a view rather different from the one I shall defend, this dynamic drives the first

1.2 Two Philosophical Methods

Although a philosophical method may itself be metaphysically neutral, a narrow focus on considerations of one sort rather than another may tend toward results of one sort rather than another. All too often philosophers talk past one another by ignoring or dismissing that which the other takes to be pivotal. In this section, I develop a pair of broad methodologies which may seem to have different consequences for questions of platonism and nominalism. The first, associated with Quine, begins with theories we already accept, and asks what there must be in order that the theories be true. Though no part of Quine's own project (!) this approach may seem associated with platonistic results, as we shall see in a couple extended examples. The second methodology begins, not with theories we already accept, but with the evidence on which those theories could be based. This may seem to be a constraint tending toward nominalism. In the end, we shall want to sort out what, if anything, is legitimate in these different approaches, and to reconcile that which remains. But first we have to say what they are.

1.2.1 Top-Down Metaphysics

Quine's method for metaphysics and his "criterion of ontological commitment" are part of the background against which contemporary metaphysics is conducted. But, perhaps for this reason, the method is often left unstated, and so understood and either adopted or resisted in different versions. Problems about its formulation lead P. van Inwagen, in "Fictional Entities" to say that "there is no proposition, no *thesis*, that can be called 'Quine's criterion of ontological commitment'" (143). He thinks that, insofar as anything deserves the name 'Quine's criterion of ontological commitment' it is a strategy or technique, not a thesis, and embraces it as such. (But it is not clear how a strategy should be less amenable to clear formulation than a thesis.) Nevertheless, I offer an account of the method and apply it in a pair of cases with platonistic results. These cases of properties and fictional objects are developed from van Inwagen, and are ones to which we shall return. I react to van Inwagen, not because I think his reasoning is flawed — quite the opposite! Van Inwagen's development is particularly

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part of Rosen, "The Refutation of Nominalism (?)" He suggests that ordinary thought is committed to "*the supervenience of the abstract... the view that the concrete array *determines* the abstract array, or in other words, that there could be no two worlds alike in all concrete respects which nonetheless differed in their inventories of abstract objects or in the relations between the abstract and the concrete which obtained in them." But "the trouble is that as soon as the picture is in place, this supervenience becomes utterly mysterious." For the problem is that in dividing the abstract from the concrete, "the picture provides a mechanism for imaging independent variation in the two domains" (153-4). Again, our project is precisely to explain how the domains are linked.

nice for our purposes insofar as he is clear-headed both about positive results, and limitations.

Quine's Method. Though Quine himself may not have been content to start this way, let us suppose there is a world and that ordinary claims and theories are true and false by virtue of the way it is. Then we get at what there is by saying what in the world makes our claims and theories true — since, if the claims and theories are true, something in the world makes them true. Faced with some true claim or theory, the conditions under which it is true may be less than obvious. But we may offer an account of that in the world which makes it true. So the ordinary claims and theories are data with metaphysical results. Quine imagines that we offer accounts of truth conditions in an extensional language along the lines of that from Frege and Russell. Given this, his approach to ontological commitments is characterized by a pair of key theses.

Quine is notorious for having stated multiple, not entirely equivalent, formulations of his first thesis. However, for our purposes, it is clear enough what he has in mind. In "On What There Is" he says, in slogan form, that "to be is to be the value of a variable" (15) and more explicitly,

Q1 A theory is committed to those and only those entities to which the bound variables of the theory must be capable of referring in order that the affirmations made in the theory be true (13-14).

The thesis has both a positive and a negative side. In "On What There Is" Quine begins by emphasizing the negative side. He thinks he is not automatically committed to objects of reference by the use of proper names, and not committed to things like meanings or properties by the use of predicates. We need not pause here, as there are problems enough from the positive part. We are positively committed to whatever things must be in the range of variables for our affirmations to be true. Thus, to take an example from Quine, if the truth condition for "some zoölogical species are cross-fertile" is, $\exists x \exists y (x$ is a zoölogical species $\wedge\ y$ is a zoölogical species $\wedge\ x \neq y \wedge x$ is cross-fertile with $y)$ then we are committed to zoölogical species as such. But if we are able to offer some other account which does not require species in the range of the variables, say, $\exists x \exists y (x$ is an animal $\wedge\ y$ is an animal $\wedge\ x \neq y \wedge x$ is not the same species as $y \wedge x$ is cross-fertile with $y)$ then so long as the account is adequate in other respects, we might be committed just to the animals required for its truth. Quine speaks of "paraphrasing the statement as to show that the seeming reference to species on the part of our bound variable was an avoidable manner of speaking" (13). But, whatever "paraphrase" may come to, we have at

least competing theories of the conditions under which the original claim is true, with divergent commitments. Observe that, so long as the truth conditions of expressions are preserved, and an argument is *valid* just in case there is no condition under which premises are true and the conclusions are not, consequences of expressions should be preserved as well — for validity hinges on nothing more than truth conditions. So a test of adequacy for this project of "regimenting" truth conditions is that consequences of expressions are retained.

We thus employ Quine's method when we move from data including theories or claims regarded as true, to accounts of the conditions under which they are true, and from such accounts to ontological commitments. I have suggested that Quine supposes accounts of truth are given in something like the extensional (canonical) notation of Frege and Russell. But this suggests a second thesis.

Q2 The truth condition for any expression (which has a truth value) may be expressed in some extensional language.

Suppose this is false, that there are true expressions whose truth condition has no expression in an extensional language. Then the situation is as follows,
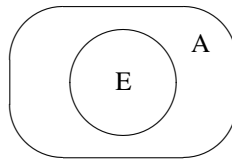
where (A) includes all expressions with a truth condition and (E) expressions with an extensional truth condition. (E) is a subset of (A). But then Q1 itself would seem to be problematic, insofar as there is room for true theories whose commitments are not those to which bound variables of an extensional language must be assigned. Faced with some recalcitrant or problematic bit of data, one might "opt out" of the method, on the ground that data falls into the range to which the method does not apply. But Q2 closes this gap. Given this, in Russell's apt phrase, opting out might seem to have "many advantages; [but] they are the same as the advantages of theft over honest toil."[9]

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[9]From a related context in "The Philosophy of Logical Atomism," (71). In a nice example of such theft, van Inwagen takes Ernest Gellner to task for saying he is a nominalist but does not try to eliminate quantification over abstract objects from his discourse; "I do not try to put what I say into canonical notation, and do not care what the notation looks like if someone else does it for me, and do not feel in the very least bound by whatever ontic commitments such a translation may disclose" ("The Last Pragmatist," 203). Against this, van Inwagen restricts himself to the observation that, "Gellner's

Q2 is particularly significant when data seems non-extensional. Extensional expressions are characterized by stability of truth value under certain substitutions. Deriving notions of singular terms, predicates and sentences from the canonical notation, let us say singular terms are *co-referential* when they pick out the same object, predicates are *co-referential* when they apply to the same objects, and sentences are *co-referential* when they have the same truth value. Then,

EX A sentence is *extensional* iff switching a singular term, predicate, or sentential part for one with the same reference cannot alter the truth value of the whole sentence.

For a stock example, suppose 'is a creature with a heart' and 'is a creature with a kidney' apply to the same individuals — where Lois believes that Superman has a heart, but being unaware of Kryptonite biology, does not believe that he has a kidney. And compare, 'Lois believes Superman is a creature with a kidney' with 'It is not the case that Superman is a creature with a kidney'. Under our assumptions, both are false. But switching the co-referential 'Superman' with 'Clark' flips the first to true, and leaves the second unchanged; switching the co-referential 'is a creature with a heart' for 'is a creature with a kidney' again changes the first to true, and leaves the second unchanged; and either switch has the effect of changing the sentential part, 'Superman is a creature with a kidney' for one with the same truth value. So the first fails each of the conditions for extensionality, and the other none. Expressions from the canonical notation all satisfy EX. But other languages might do so as well. Certain fragments of ordinary language are extensional, as might be expressions of languages with plural quantifiers, or infinitely long expressions. (Both plural quantification and infinitary languages are taken up in the following. Presentations for infinitary logic get technical quickly; however Nadel, "$\mathcal{L}_{\omega_1\omega}$ and Admissible Fragments" and Dickmann, "Larger Infinitary Languages" are a reasonable gateway. Oliver and Smiley, *Plural Logic* develops the other.)

Quine might justify Q2 as a theoretical or experimental result: as a matter of fact, when we set out to give truth conditions for the sentences of science or whatever, we find that expressions of the canonical notion, or of some (maybe extended) extensional language suffice. On the face of it, this suggestion is implausible — or, at least, Quine uses the fact that supposed truth conditions for certain expressions have no adequate extensional account as reason for questioning the intelligibility of those

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confession comes down to this: I don't mind contradicting myself — I don't mind both saying things that imply that there are abstractions . . . and saying that there are no abstractions — if figuring out how to avoid contradicting myself would require intellectual effort" ("Fictional Entities," 144).

sentences. Thus in his classic, "Reference and Modality" he reasons from problems of substitutivity in modal contexts to his conclusion that results are "so much the worse for quantified modal logic" and "for unquantified modal logic as well" (156). The reasoning is hardly uncontroversial! My point here is just that data from modal logic, together with the negative results for extensionality, would seem to be just the sort of evidence that should call an experimental justification for Q2 into question.

But there may be other reasons to accept Q2. Quine begins "Reference and Modality" linking sameness of truth value on substitution of singular terms to the indiscernibility of identicals. By the indiscernibility of identicals, if $a = b$, the properties of a are the same as the properties of b. So, given a picture of language as picking out things and saying that they have whatever features they do, from $a = b$ and Fa we expect Fb. We have assumed that there is a world and that sentences are true or false by virtue of the way it is. Without prejudicing the question of what sorts of things there are, and of the nature of their properties or relations, one might think that this assumption amounts to saying that there are things with properties and relations, and that it is the things with their properties and relations that make sentences true or false. Say this is right. Then all we need to describe the world is the ability to pick out things and to say that they have whatever properties and relations they do. It may be that the truth condition for some sentence is that things or the world satisfy some complex condition; but so long as we are able to state this condition, there is nothing more to be said about the world. This seems to be just the sort of thing extensional languages are fitted to do. And it motivates Q2.

Now consider what happens when extensionality seems to fail. "Reference and Modality" opens with the point about the indiscernibility of identicals, and then an example where substitutivity seems problematic. Based on the true identity, 'Cicero = Tully', we might substitute into,

'Cicero' contains six letters

to obtain, "'Tully' contains six letters." But the former is true and the latter is false.[10] However this is not held to be a failure of substitutivity, insofar as nothing in the displayed sentence names Cicero. Rather, what is named is a word that contains six letters. A proper substitution is based on "'Cicero' = Tully's other name that begins with 'C'", to reach the perfectly true, "Tully's other name that begins with 'C' contains six letters." By requiring that our language be extensional, we thus require clarity

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[10]In a dismal episode from graduate school, I was trying to reproduce this example on a qualifying examination. Unfortunately, and typically, I misspelled Cicero's other name 'Tulley'. But then it did contain six letters! Flummoxed, I ended up creating an entirely separate example to make the point. Despite this sort of thing, I somehow managed to pass.

about what things are said to have what properties. And this is just right when we are concerned about questions of ontology.

More substantively, suppose we let a language have an operator for ⟨F⟩ for "at some future time" and that as it happens, 'is Bob's brother' and 'is Bob's sibling' are co-referential, although Bob has a sister on the way. In this case, ⟨F⟩∃x(x is Bob's sibling ∧ x is female) is true though ⟨F⟩∃x(x is Bob's brother ∧ x is female) is false. So truth value flips upon substitution of co-referential terms and extensionality fails. A response, like the one above, is that we have not gotten the objects and properties right. Thus, ∃t∃x(t is a time after now ∧x is Bob's sibling at t ∧ x is female at t) is true, while ∃t∃x(t is a time after now ∧ x is Bob's brother at t ∧ x is female at t) is false. But there is no failure of extensionality, insofar as there is a sister and time to which 'x is Bob's sibling at t' applies but 'x is Bob's brother at t' does not; so the relations are not co-referential, even though they apply to the same persons now. Of course, among the objects over which the quantifiers range are times.

Little changes if the we consider an operator ◇ for possibility where ◇∃x(x is Bob's sibling ∧ x is female) is true, though ◇∃x(x is Bob's brother ∧ x is female) is false. Where 'x is Bob's sibling' and 'x is Bob's brother' actually apply to the same individuals, extensionality fails. One response is to let the quantifiers range over *worlds*. Thus, ∃w∃x(w is a way the world could be ∧ x is Bob's sibling at w ∧ x is female at w) is true, while ∃w∃x(w is a way the world could be ∧ x is Bob's brother at w ∧ x is female at w) is false. But there is no failure of extensionality, insofar as there is a sister and world to which 'x is Bob's sibling at w' applies but 'x is Bob's brother at w' does not; so the relations are not co-extensional, even though they overlap in actuality. And we quantify over ways the world can be.

Of course these are not the only responses. The appeal to other worlds is particularly problematic, and we shall be interested in alternatives in what follows. Quine is satisfied enough with the response for times, and with corresponding ontological commitments. But he is not at all happy with the appeal to worlds. Our point for now is simply to emphasize the way Q1 combines with Q2 to pressure the way we think things are. Within the method, one is pressed to (i) accept the commitments, (ii) reject the data, or (iii) offer an alternative account of the truth conditions. In the ordinary case original data is secure — more secure than philosophical theories proposed to account for it. So we are engaged in the project of offering theories to account for truth, and so to identify corresponding ontological commitments.

Fictional Objects. In a series of articles spread over some 25 years, van Inwagen applies Quinean criteria for the conclusion that some things are fictional characters.[11] Van Inwagen does not think Tolkien, for example, says something true or false when he says, in the first line of *The Lord of the Rings*, "When Mr. Bilbo Baggins of Bag End announced that he would shortly be celebrating his eleventy-first birthday . . . there was much talk and excitement in Hobbiton." It would not make sense for someone peering over Tolkien's shoulder when he was writing this to say, "How true!" or, "No, no, you've got it all wrong, that's false!" (cf. "Creatures of Fiction," 41). However expressions of the sort,

> There are characters in some nineteenth-century novels who are presented with a greater wealth of physical detail than is any character in any eighteenth-century novel.

have truth values. That is, expressions of what we might call, "literary criticism" *about* fiction have truth values. As such, they are objects to which Quine's method applies. So, a natural proposal is that the above expression has a structure of the sort, $\exists x(x$ is a character in a nineteenth-century novel $\wedge\ \forall y[y$ is a character in an eighteenth-century novel $\rightarrow x$ is presented with a greater wealth of physical detail than is $y])$. Among the things that must be assigned to the variables for this to be true are some characters in nineteenth-century novels. So by Q1 we are so-committed — or we are so-committed unless we want to reject the original data (and all other data of the sort), or can provide an alternative account which does not require the existence of the characters.

Van Inwagen is not content to reject the data, and does not think plausible alternative accounts of truth are forthcoming. So he accepts the commitment. It is difficult to demonstrate that no adequate alternative is possible. However van Inwagen emphasizes the constraint on such accounts mentioned above ("Creatures of Fiction," 45f). One might think that a commitment-free alternative is readily available. Seemingly, it is *novels*, or classes of novels, that do the work; so why not introduce a special relation, say, '*x dwelphs y*' which applies iff members of x include a certain sort of "character development" more detailed than any in members of y. Then we say, as an account of the above, 'the class of nineteenth-century novels dwelphs the class of eighteenth-century novels' — and restrict our commitment to classes of novels. Presumably we get the truth values right. But van Inwagen asks us also to consider also,

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[11]"Creatures of Fiction," "Fiction and Metaphysics," "Pretense and Paraphrase," "Quantification and Fictional Discourse," and "Fictional Entities."

> Every female character in any eighteenth-century novel is such that there
> is some character in some nineteenth-century novel who is presented with
> a greater wealth of physical detail than she is.

This is an obvious consequence of the original. And it follows also from the account of truth which quantifies over fictional characters. We might attempt an account that avoids characters as before — perhaps, "the class of nineteenth-century novels praphs the class of eighteenth-century novels." But there is no obvious reason to think this follows from the claim that the class of nineteenth-century novels dwelphs the class of eighteenth-century novels. So the proposed alternative does not preserve logical consequence, and so far fails a condition of adequacy. Van Inwagen thinks accounts which preserve consequences "almost certainly . . . will have a quantificational structure not much simpler than the (apparent) quantificational structure of its 'original'." Thus it is likely that commitments will remain.

Supposing that commitments remain, and so that there are creatures of fiction, there is the question what they are like. Presumably no physical things are hobbits, and no physical place is Middle Earth. But van Inwagen distinguishes properties fictional characters *have* from ones they *hold* (*in a text or location*). Fictional characters have logical properties like BEING SELF-IDENTICAL, along with literary properties like BEING CREATED BY TOLKIEN or BEING A CHARACTER OF BOTH PRINT AND FILM. They hold properties of the sort, BEING SHORT, HAVING HAIRY FEET, and LIVING IN MIDDLE EARTH. Thus it is van Inwagen's view that "what appears to be the apparatus of predication in 'fictional discourse' is ambiguous. Sometimes it expresses actual predication, and sometimes an entirely different relation" ("Fictional Entities," 148). When we say, "Frodo was created by Tolkien" we say that a certain property is possessed by the fictional character. When we say "Frodo is a hobbit" we say not that the character has, but that it stands in the holding relation to a property. So there is no pressure find short physical things with hairy feet to account for the truth of simple claims of the sort, 'Frodo is a hobbit'.[12]

As van Inwagen observes, other theories may have the same general structure of allowing that there are fictional characters, with a distinction between properties the characters have, and ones they "hold." He candidly agrees that he does not have much to say about the nature of characters such that they have some properties and hold others — and brings this out by comparison to the parallel but more specific theories of Wolterstorff and Thomasson. On Wolterstorff's view, fictional characters are certain

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[12]And similarly, 'Frodo does not exist' might be a way of saying truly that nothing has all the physical properties that Frodo holds. Or, for someone truly lost in the story, that 'Frodo' is not the name of a physical thing, but rather is a name held by a certain fictional character ("Fictional Entities," 146-7).

kinds, where kinds are platonic entities (*Worlds and Works of Art*). Very roughly, say one kind *entails* another iff it is not possible for something to be of the first but not the second; then where properties are parallel to kinds, a character holds a property just in case it entails the property. This view has the consequence that fictional characters are eternal beings (or no less eternal than kinds), with all their properties essentially, discovered rather than created by their authors. In contrast, Thomasson holds that fictional characters are contingent beings, 'abstract artifacts' created by their authors (*Fiction in Metaphysics*). A distinction like having and holding remains, but the nature of the objects is quite the opposite from the ones posited by Wolterstorff. Van Inwagen observes that these theories are subject to various concerns which he avoids "by the clever expedient of being vague" ("Fictional Entities," 152). We shall return to this point shortly, when we turn to consideration of Problems About Primitives.

Properties.   Reasoning is parallel in van Inwagen's, "A Theory of Properties." So Quine's method, backed by entailment considerations, has the result that there are properties. But the account of what they are turns out very thin. Van Inwagen takes as his example,

> Spiders share some of the anatomical features of insects

This has the apparent form, $\exists x(x$ is an anatomical feature $\land$ insects have $x \land$ spiders have $x)$, which is true only if something in the range of the variables is an anatomical feature. But features, qualities, characteristics, properties, and the like may seem to be just the same thing. (And if there are distinctions to be made between any of these, it is likely that one will be no more palatable to the nominalist than another.) So by Q1 there is an apparent commitment to properties — unless, of course, we are willing to reject the data, or there is an acceptable way to account for the data that avoids the consequence.

In this case, the data seems secure — there is no denying that spiders share some of the anatomical features of insects. Again, it is impossible to survey all the attempts to account for the data. But the difficulty of providing alternatives is highlighted by the constraint that accounts of truth conditions preserve logical consequence. Consider a (reasonably traditional) response along the following lines: spiders share some of the anatomical features of insects just in case spiders are like insects in some anatomically relevant ways. We require a resemblance between spiders and insects. But van Inwagen observes that this seems to require a quantification over "ways one thing can be like another," something like, $\exists x(x$ is a way one thing can be like another $\land$ x is anatomically relevant $\land$ spiders are like insects in $x)$; thus there is commitment to ways one thing can be like another. And this may seem to be a

perversion of the resemblance strategy. Perhaps the idea is merely to observe that there is an unstructured relation between the class of spiders and the class of insects so that, say, the class of spiders *blaphs* the class of insects. So far, so good. But consider the argument,

1. If two female spiders are of the same species, then one is like the other in all anatomically relevant ways.

2. If *a* is like *b* in some anatomically relevant way, and *b* is like *c* in the same way, then *a* is like *c* in that way.
   ____

3. An insect that is like a female spider in some anatomically relevant ways is like any female spider of the same species in some anatomically relevant ways.

The argument is valid. And if the premise and conclusion are given a structured account, then the conclusion follows by the usual methods. (Challenge: try it!) But it is hardly clear how the conclusion results where the premise and conclusion are unstructured. The point here is exactly parallel to the one for fictional characters. In effect, there is a bulge in the carpet: chasing it around moves the quantification from one place to another. In the end, van Inwagen thinks, the most comfortable place will be to rest where we started, with quantification over properties, and so with commitment to properties.

When it comes to saying what these things are like, van Inwagen offers a theory which, he says, is "nearly vacuous" (131). If we set out to describe the intrinsic nature of a pen or the like, we will have a great many things to say — about the nature of the ink, the working of the ball, or whatever. But not so for abstract objects in general, and properties in particular. Van Inwagen does, however, lay out a certain *role* which is at least inconsistent with some things others have had to say about properties. His idea is to identify the property role with the role, "thing that can be said of something." Some things can be said — as that Chicago has a population over two million. But this cannot be said of anything. However other things can be said of a thing, as of Chicago, or New York, or (falsely) of South Bend, that it has a population over two million. He calls things that can be said, or said of things *assertibles*; assertibles that can be said of things are *unsaturated*; and he applies the usual logical operations to these. Thus "that it has a population over two million" and "that it once filed an income-tax return" are unsaturated assertibles, as is "that it either has a population over two million or has filed an income-tax return." We seem to quantify over unsaturated assertibles very much as we do properties as in, "all the negative things you've said about Gore are

perfectly true, but don't you see that they're equally applicable to Bush?" (132). And it seems natural to identify unsaturated assertibles with properties.

But, so far, the picture is subject to paradox. If there are unsaturated assertibles, then things can be said of them. Among the things we can say of "that it is white" for example, is that it is not white — the assertible is not a thing of the sort that has a color. So "that it is white" cannot be truly be said of itself. And similarly for many things that can be said of something — thus "that it has a population over two million" does not have a population at all, and cannot be truly said of itself. So, seemingly, one of the things that can be said of a thing is "that it cannot be truly said of itself." But now there is trouble: Suppose this thing, "that it cannot be truly said of itself" is such that it can be truly said of itself; then for this reason it cannot be truly be said of itself. So suppose it cannot be truly be said of itself; then for this reason it can be truly said of itself. And this is impossible. Van Inwagen chooses to deny that one of the things that can be said of things is "that it cannot be truly said of itself." As he observes, the existence of such an assertible may seem self-evident. However, so does the existence of a set whose members are just those sets that are not members of themselves. And anyone who accepts that there are sets will have to accept some response to Russell's paradox. (Is the set of all sets not members of themselves a member of itself or no?) There are a variety of solutions — none, perhaps ideal. But whatever the solution may be, "the friends of things that can be said of things can easily adapt any of the standard, workable ways of dealing with the paradox to the task of saying which open sentences must correspond to things that can be said about things" (134).

Supposing, then, a reasonable response to the problem of paradox, the theory is left with some interesting consequences. Properties are not parts or constituents of concrete objects. Physical atoms and the like may be parts of a thing, but things that can be said of them are not among their parts. For this reason, properties are not "somehow more basic ontologically than the objects whose properties they are" (135). If properties are things that can be said of something, then among the properties are ones that are complex like SINGING IN VIENNA OR MADE OF ICE (Michael Jubien's favorite property), so that properties are "abundant" rather than "sparse." And among the properties are ones that are uninstantiated like BEING ROUND OR NOT ROUND. Given these consequences (and more), van Inwagen remains concerned about the lack of content about their intrinsic nature. "The fact that this theory is inconsistent with various interesting and important theses about properties shows that, although it may be very close to being vacuous, it does not manage to be entirely vacuous" (138). Again, we shall return to this point, when we turn to consideration of Problems About Primitives.

Insofar as the Quinean motivation yields the existential claims about fictional objects, properties and the like apart from content about their intrinsic nature, it may

seem so far neutral between hard and soft platonism. To the extent that nominalist theories of either sort fail to account for data about fictional characters, properties or the like, the platonist is likely to hold that nominalism simply fails. But, from a different starting point, the nominalist is likely to think the same about platonism. As we shall see, from the perspective of another methodological starting point, what counts as a compulsory question, and so the proper location of the burden of proof, may very well differ.[13]

1.2.2 Bottom-Up Metaphysics

Nominalism may seem motivated on the Ockhamist ground that, other things being equal, it is better to omit abstract objects than to include them, or better to omit them apart from the "necessity" of letting them in. No doubt there is something right about this. The problem is that other things are so complex as rarely to be equal, and the occasion of such necessity is hardly clear — thus the Quinean arguments. But there is more than this to the motivation for nominalism. Though it may not have been his purpose, motives for nominalism are often driven by an epistemological argument drawn from Benacerraf, "Mathematical Truth." In its typical form, this is presented as an argument for rejecting platonism. But on its flip-side, it is an argument for accepting nominalism. The reasoning is difficult to pin down, though it has been persuasive, and I think is best seen as pointing toward a methodology pressing in the direction of nominalistic results. In the following, I develop this reasoning, and turn to a couple of extended examples.

The Epistemological Argument.   As indicated by his title, "Mathematical Truth," Benacerraf focusses on mathematical entities. And, though it seems forgotten in the many references to his article, he begins by noting the desirability of a uniform account of truth and inference for expressions like, "there are at last three large cities older than New York," and "there are at least three perfect numbers greater than 17" (a perfect number is equal to the sum of its factors). But the only such account we have, says Benacerraf, is the referential one with the natural consequence that there are abstract numbers, and so that some form platonism is true. So far, all this runs the Quinean machine. But the problem is that the result seems incompatible with natural assumptions about knowledge. The difficulty might be put as follows.

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[13]This point about burden of proof is a theme of the first section from Burgess and Rosen, *A Subject With No Object*.

1. If platonism is true, then we are causally isolated from abstract numbers.

2. If we are causally isolated from some things, then we cannot have knowledge of them.

3. We can have knowledge about numbers.

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4. Platonism is not true.

As stated, the argument falls into the pattern: if A then B; if B then C; not-C; so not-A. So it is valid. The third premise is just the datum that mathematical knowledge is possible — the point of the argument is that, whatever mathematical knowledge is, it is not knowledge of abstract entities from which we are causally isolated. And the reasoning may be generalized for abstract objects of other kinds; given their supposed nature, it is mysterious how knowledge of them is possible. But the first two premises are neither clear nor obvious.

Much discussion centers on the second premise. Benacerraf says he favors a "causal account of knowledge on which for X to know that S is true requires some causal relation to obtain between X and the referents of the names, predicates, and quantifiers of S" (22).[14] A causal theory of knowledge was proposed by Goldman, "A Causal Theory of Knowing" (and others) in response to concerns about the analysis of knowledge from Gettier, "Is Justified True Belief Knowledge?" The problem is that one may have a belief that is justified and true that nonetheless falls short of knowledge — for there may be something lucky or coincidental about its being true. Thus, for a standard example, suppose I see Smith driving a Ford, she offers me a ride to work in the Ford, and so forth. On this basis, I form the belief that Smith owns a Ford. Given this experience, my belief is justified. Further, it is true that Smith owns a Ford — only hers is a classic Thunderbird housed in her garage, and the Ford I have seen and in which I have ridden is borrowed from her brother-in-law. So I have a justified true belief that Smith owns a Ford, but its base is defective in a way that makes it fall short of knowledge. The idea of causal theories is to require an "appropriate" causal connection between a belief and what is believed. In this case, knowledge fails insofar as the justified true belief is not connected to the Ford Smith in fact owns.

As it happens, such theories have been subjected to a host of objections. Benacerraf means to rule out knowledge of abstract objects by means of a causal constraint; so what matters for his argument is whether causal connections are *necessary* for

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[14]In addition, he favors a causal theory of reference. I follow Benacerraf, and tradition, in emphasizing the epistemological line. For discussion of the referential consideration, see Burgess and Rosen, *A Subject With No Object*, 49-60.

knowledge. For a caution relevant to our concerns, suppose one were to adopt a view according to which medium-sized physical objects do not enter into causal interactions at all — that all causal interactions take place at the microphysical level. (Though this is not his view, perhaps it is inspired by something like the "exclusion argument" from Kim's, *Mind in a Physical World*.) Ordinary physical objects supervene in the sense of ST on things at the microphysical level. Given this view, though there are no causal interactions among ordinary physical things, surely we would want to hold that knowledge of them remains possible — perhaps by causal interactions with entities on which they depend. This might be a relevant analogy for soft versions of platonism, on which abstract objects depend on ones that are concrete. At any rate, a direct causal connection of the sort contemplated by the causal theory seems too strong; for there might be sorts of dependence other than direct causal dependence sufficient to satisfy requirements for knowledge.[15]

In fact, problems with the causal theory have not deterred philosophers from thinking there is something right about Benacerraf's basic strategy. Thus Field puts the problem as a challenge to explain the reliability of beliefs about abstract numbers in his, *Realism, Mathematics and Modality* (25-30, 230-239). Field is "not optimistic" about the prospects of providing the required explanation. Unless explanations are supposed to include some mechanism to make beliefs responsive to their objects, however, such a requirement strikes me as too weak. To see this, consider strategies that would attain reliability by mere "harmony." Suppose that as a result of ingesting some drugs, for any type of marble, I hallucinate that there is one in some jar. When asked if there is a marble with red stripes in the jar, I respond "yea man, goovy," and similarly in every case. In fact there is a marble with red stripes in the jar, for the jar contains marbles of every possible variety. The nature of reality together with my method explains the reliability of my descriptions. But surely, apart from some story about common cause or other linkage, I do not know that the jar contains a marble with red stripes — for the reliability is itself coincidental or lucky.

The case is not that far out. Thus compare Balaguer, *Platonism and Anti-Platonism in Mathematics*, who sets up the challenge along Field's lines, and defends a view on which there are mathematical objects of all logically possible kinds; so that when mathematicians propose theories about any objects that are logically possible, they reliably describe aspects of abstract reality. Or again, consider Lewis's view from *On The Plurality of Worlds*, on which for every way a world can be, some world

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[15]For another sort of case, Alice becomes aware that Jones has ingested a fatal dose of poison and that there is no antidote, and so comes to know that Jones is dead. But, in fact, Jones dies from a bullet to the head, before the poison takes effect. In this case, it appears that the causal links do not reach from Alice to the death, even though her belief is more than mere luck or coincidence.

is. On this view, worlds are things like the universe in which we live, with concrete parts, but different worlds are spatiotemporally and causally isolated. Thus, as Lewis acknowledges, the Benacerraf strategy has seeming application to his view — and may account for something of the "incredulous stare" with which Lewis says his view is confronted. Of course, one man's modus ponens is another's modus tollens: Lewis sets up the objection, and proceeds to use his view along with the case of mathematics to reject a causal requirement (108-115). We have *a priori* knowledge that there could be a talking donkey. Given this, with the hypothesis that for every way a world could be some world is, it follows that there is a world in which there is a talking donkey. So the harmony between worlds and modal method results in true beliefs. To the extent that mere harmony is not sufficient for knowledge, it is natural to think that some substantive link between a belief and its objects is required — is part of what is required to eliminate lucky true belief.[16]

Though Benacerraf declares allegiance to the causal theory of knowledge, in fact he needs, and applies, the causal constraint as a sort of minimum condition.

> In brief, in conjunction with our other knowledge, we use p to determine the range of possible relevant evidence. We use what we know of X (the putaive knower) to determine whether there could have been an appropriate kind of interaction, whether X's current belief that p is causally related in a suitable way with what is the case because p is true — whether his evidence is drawn from the range determined by p. If not, then X could not know that p. (24)

Granting problems with the specifically causal constraint, it seems reasonable that a person cannot come to know something entirely independent of evidence on which is known — on pain of the charge that belief, if true, is true by luck or coincidence. Thus there would seem to be room for a version of the second premise not tied directly to the causal theory of knowledge, but rather to some more general notion of dependence.

This brings us back to the first premise according to which platonism requires causal isolation from abstract numbers. If we accept a modified version of premise two and the argument is to remain valid, the first premise requires parallel modification. And this returns us to tangled questions about distinctions between hard and soft platonism, and the nature of dependence between abstract and concrete things. Rather than work with the argument as it stands, I think we can make progress by recasting the reasoning for the positive result that nominalism is true.

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[16]And the point need not be a reversion to internalist as opposed to externalist theories of justification, as Balaguer seems to suggest (53-58). As in the case of the causal theory, a request for something beyond mere regularity need not be a request for something internal.

1. Data for our theories is empirical.

2. If data for our theories is empirical, then the reasonable range of our theories is restricted to ones whose objects are somehow dependent on concrete objects.

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3. The reasonable range of our theories is exhausted by nominalistic theories.

From (1) and (2), the reasonable range of our theories is restricted to ones whose objects are somehow dependent on concrete objects. But SN just is the thesis that all objects are grounded in, and so dependent upon, ones that are concrete. So the result seems sufficient at least to restrict the reasonable range of our theories to those that fall under the scope of soft nominalism. The result turns to hard nominalism with the additional premise that everything dependent on concrete objects *is* concrete.

At one stage, Plato held a doctrine according to which some data for our theories is not empirical: in this life we recollect a sort of "communion" with the forms from before we we were born (*Meno*, 81). This would seem to undercut (1). Similarly, in one place Gödel seems to advocate a direct perception of mathematical objects: "Despite their remoteness from sense experience, we do have a perception also of the objects of set theory, as is seen from the fact that the axioms force themselves upon us as being true. I don't see why we should have less confidence in this kind of perception, i.e., in mathematical intuition, than in sense perception" ("What is Cantor's Continuum Problem?" 483-4). Thus, again, there might be a sort of data other than ordinary empirical data. But suppose we set these proposals to the side, and focus just on theories which *are* based on ordinary empirical data. Then the issue shifts to (2).

The second premise brings us back to the questions about the sort of dependence that is required between objects and evidence for knowledge, and the sort of dependencies that may or may not obtain between abstract and concrete objects. But whatever the linkages are supposed to be, it may be that hard platonism at least is sufficient to rule out whatever is required for knowledge. In this case, the argument is sufficient to establish it's conclusion, insofar as the conclusion is just that reasonable theories are nominalistic in one or the other sense. Thus the nominalist offers a method that begins, not with completed theories, but with empirical data on which theories are based. Beginning with such data, results are limited to theories in a certain range. Insofar as a theory has consequences incompatible with nominalistic results, the theory exceeds its legitimate range — legitimate consequences of theories justified by empirical data do not extend to absolutely independent abstracta. Of course, apart from a premise

according to which everything grounded in the concrete is concrete, this does not so far rule out grounded platonism of the sort I mean to defend.

It is impossible to represent all the nominalistic strategies by a pair of limited examples. As we shall see, however, it is no simple matter to carry this nominalistic project through.

Modal Fictionalism. From Quine's method, short of opting out, when faced with some data that seems to have whatever ontic commitments, we have seen three options: accept the commitment; give up the data; offer an account of truth without the commitments. But there is another path, taken on occasion by Quine himself: show that the original expressions are not offered as serious and literal truth about the world. It may have been Quine's view that when we say things that are not the serious and literal truth about the world, we should eventually give them up in favor of things that are. Then we are back to the first three. But one can have reasons to *retain* claims which are not seriously and literally true. This is the idea of fictionalism. So, Kendall Walton (*Mimesis as Make-Believe*, "Metaphor and Prop Oriented Make-Believe") speaks of *props* which play roles in games of *pretense* — as a stick may be be a horse, or a stump a bear, in a child's game of make believe. In these cases, the props function in service of the make believe. But on occasion make believe is harnessed in the understanding of props. Thus,

> Where in Italy is the town of Crotone?, I ask. You explain that it is on the arch of the Italian boot. 'See that thundercloud over there — the big angry face near the horizon', you say; 'it is headed this way'.... All of these cases are linked to make-believe. We think of Italy and the thundercloud as something like pictures. Italy (or a map of Italy) depicts a boot. The cloud is a prop which makes it fictional that there is an angry face.... But our interest, in these instances, is not in the make-believe itself, and it is not for the sake of games of make-believe that we regard these things as props.... [Make believe] is useful for articulating, remembering, and communicating facts about the props — about the geography of Italy, or the identity of the storm cloud. ("Metaphor and Prop Oriented Make-Believe," 66-7)

A great deal of complex information about the cloud may be coded into the claim that there is an angry face near the horizon — not easily put in a literal way. It remains useful to retain the claim about the face, though none of the options (i) - (iii) apply. In particular, in the context of the pretext, there is no commitment to anything like a face

on the horizon. Rather, the rules of the make believe map our claim about the face onto useful content about the world.[17]

But it might be thought that talk of numbers, properties, possible worlds and the like is itself similarly a sort of pretext that should carry no ontological commitment. As an example of *fictionalism* as a nominalistic strategy, I briefly consider Gideon Rosen's,"Modal Fictionalism." We have seen from Quine's method a pressure to let claims about what is *necessary* range over *ways the world could be* (p. 16). In *On The Plurality of Worlds*, Lewis allows that there are such things, and that they are very much like the universe in which we find ourselves — only causally and spatiotemporally disconnected from the world in which we live. Other philosophers hold that such claims range over abstract entities which, in some sense, *represent* ways the world could be (for example, Plantinga, *Nature of Necessity*, van Inwagen, "Two Concepts of Possible Worlds"). Rosen suggests that Lewis's proposal in particular is a good one, and that we can retain the benefits without the costs by regarding it as pretense. Thus from worlds theory, necessarily P iff P is true at every possible world, and possibly P iff P is true at some possible world. More generally, there is a wide range of notions, including strict and counterfactual conditionals, and the like, which may be analyzed in terms in worlds. Thus a modal proposition P will have some analysis P^* in terms of worlds. Then the modal fictionalist says, simply, necessarily P iff according to the fiction of possible worlds P is true at every possible world, and possibly P iff according to the fiction of possible worlds P is true at some possible world. And, more generally,

MF P iff according to the fiction of possible worlds, P^*.

Thus Rosen's fictionalism is parasitic on Lewis's realist account. For any analysis Lewis offers, the fictionalist offers one like it based on the fiction of possible worlds. The idea is that the fiction supplies a set of rules which convert make believe claims about worlds into substantive claims about possibility and necessity.

This proposal is subject to a number of technical objections with technical replies.[18] I make a couple of philosophical observations which set up discussion to come. First, Daniel Nolan, "Three Problems for 'Strong' Modal Fictionalism" and Seahwa Kim, "Modal Fictionalism and Analysis" develop a problem according to which the eternal and necessary character of modal truth is incompatible with the

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[17]The discussion of this paragraph draws on Yablo, "Go Figure," and "Does Ontology Rest on a Mistake?" together with Walton.

[18]For difficulties, see Brock, "Response to Rosen," Hale, "A Simple Dilemma," Divers, "Modal Fictionalism Cannot Deliver Possible Worlds Semantics," and, for helpful overall discussion with further references, Nolan's entry, "Modal Fictionalism," in the *Stanford Encyclopedia of Philosophy*.

mind-dependence of fictions about possible worlds. Thus, not so long ago there was no fiction about possible worlds, the fiction of possible worlds might not have been told at all, and given that it has been told, might have been told in different ways. But then, from MF, modal truths vary along with the fictions — and this seems wrong. Though different replies may be possible, both Nolan and Kim end up suggesting a "rigidifying" response on behalf of the fictionalist. One avoids variation with alternate fictions, by avoiding alternate fictions. Thus, one might suggest, MF′ *P* iff according to *Lewis's actual* fiction of possible worlds, *P**. The proposal avoids the result that modal truths themselves vary — for they are consistently linked to a single fiction. As Kim observes, it remains that modal statements are true in virtue of a *contingent* story. However, he holds that the fictionalist should accept the consequence. "It is true that there is something bizarre in the thought that modal statements are true in virtue of the content of some fiction which might not have existed and didn't exist a long time ago. But although it is bizarre, it is still the case that modal statements are true in virtue of the content of some fiction" (130).[19] This point will matter again shortly, when we turn to consideration of grounds for modality, and Problems About Primitives.

We get to a related point by consideration of a worry Rosen himself raises about the completeness of the modal fiction. When Rosen sets out to tell the possible worlds fiction he includes Lewis's claim that there is a plurality of worlds. He includes an "encyclopedia" with all non-modal truths about the actual world, and a principle of recombination according to which parts of universes combine in arbitrary ways (provided there is a spacetime large enough to hold them). But he worries that modal truths on the story so told might come apart from modal truths on Lewis's account. Thus Lewis says there is a fact of the matter about the maximum "size" of possible worlds — about the maximum κ of non-overlapping physical objects in a single world, for cardinal number κ. But this maximum is unknown, and nothing in the fiction of possible worlds says that there is a world containing κ non-overlapping physical objects or that there isn't. Since the fiction does not say there is a world containing κ non-overlapping physical objects, by MF it is not the case that there could have been κ non-overlapping physical objects. But this is to decide what seems to have been an open question in the negative, given just the incompleteness of our story — itself driven by ignorance. It seems more natural to say something like MF″ *P* iff according to Lewis's actual fiction of possible worlds, *P** — unless the fiction is silent about *P** in which case *P* is truth-valueless (for such a proposal, see Nolan,

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[19]This depends on the distinction between *strong* and *timid* fictionalism (see Rosen, "Modal Fictionalism," 354). The strong fictionalist looks to the fiction for an account of modal truth, where the timid fictionalist merely uses the fiction to translate to and from talk of worlds. In the latter case, there need be no such strangeness.

"Modal Fictionalism"). Even so, the fictionalist leaves undecided that which is decided on Lewis's account, so that the views come apart.

And problems of incompleteness are broader than this. Very little about non-actual worlds is included *explicitly* in the story of possible worlds. There is the encyclopedia of non-modal truths about the actual world, and there is the principle of recombination. But, to take an example from Rosen (348 n27), it is hardly obvious whether truths about the actual world together with principles of recombination will have the result that according to the fiction of possible worlds gold is a metal in every possible world. This puts a lot of weight on the operator, 'according to the fiction of possible worlds'. Rosen offers as glosses, *if the fiction were true, P* would be true*; *if we suppose the fiction is true, P* follows*; *it would be impossible for the fiction to be true without P* being true as well*. But these notions are themselves modal. This is a problem if the fictionalist theory is to be an account of modal notions. Officially, Rosen counts the story operator as a primitive of his theory — though, unless and until something better comes along, he grants that it is a modal primitive.

In fact, there is a sort of trade off between the operator, and what is explicitly in the story. At the one extreme, stories may be very short, with all the content in the operator: Insofar as the operator builds in whatever is required for modal truth, the operator is itself sufficient for the complete story about worlds. There may be epistemic questions about its application, but this does not undercut the fullness of the story which follows from the range of modal truths built into the operator. At the other extreme, one can pump up explicit content of the stories so that the story operator merely reports, and so is not modal at all. In this case, the content of stories far exceeds anything we humans are likely to say — including the disposition of every point of spacetime for every way the world could be. Proposals along these lines push stories in the direction of abstract entities — and so might seem to merge with accounts of the sort nominalist fictionalism meant to avoid. I think Rosen means his story operator to be somewhere in the middle; as indicated above, it is to be a *consequence relation*.[20] But it is hardly clear that this is stable ground. Thus, for example, Lewis presses the point that there are "microreduction laws" that express necessary truths connecting the micro- and macro-realms (*On The Plurality of Worlds*, 155-6, see also Sider, "The Ersatz Pluriverse"). It is hardly obvious, for example, whether some micro-combinations allow for green gold. But nobody knows what

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[20]This makes sense of the incompleteness concern from the paragraph above. Observe that the problem goes away at the endpoints: On the one hand, insofar as there is a modal fact of the matter, the story operator can simply *build in* that there is, or is not, a world with κ non-overlapping physical things. And on the other, each world can explicitly represent the number of non-overlapping physical things it has.

these laws are; so they are not part of the stories — or they are not unless we slide one way or the other toward the extremes. Again, we will return to this, when we turn to consideration of grounds for modality and Problems About Primitives. At least the fictionalist story cannot be the end of the matter for the nominalist.

Modal Structuralism. In one place, Quine observes that "classical mathematics, as the example of primes larger than a million clearly illustrates, is up to its neck in commitments to an ontology of abstract entities" ("On What There Is," 13). It is often observed that accounts for arbitrary mathematical objects may be offered in terms of sets. Thus on Zermelo's account, 0 is the empty set ϕ, and the successor of any n is the set containing n and all its members; so the positive integers are, $\{\phi\}, \{\phi, \{\phi\}\}, \{\phi, \{\phi\}, \{\phi, \{\phi\}\}\}\ldots$. Other entities, rationals, reals, and the like, are constructions out of these. But, as Benacerraf observes in another famous article, "What Numbers Could Not Be," there are more than one ways to skin this cat. So, Zermelo's is not the only account of the integers; for von Neumann, 0 is the empty set, and the successor of any n is the unit set $\{n\}$; so the positive integers are, $\{\phi\}, \{\{\phi\}\}, \{\{\{\phi\}\}\}\ldots$. The important point, says, Benacerraf, "is not the individuality of each element but the structure which they jointly exhibit" (290). And insofar as each exhibits the essential structure in which one member follows the next, one is as good as the other. Benacerraf's point is a leading idea behind *structuralism* in mathematics; and the approach is taken in a nominalistic direction in the very interesting work of Geoffrey Hellman (see his, *Mathematics Without Numbers*, "Structuralism Without Structures," and nice review article "Structuralism" in *The Oxford Handbook of Philosophy of Mathematics and Logic*). I sketch broad outlines, which should suffice for our purposes.

In second-order logic, one allows quantifiers to bind *predicate* as well as individual variables. Thus, $\forall x \forall y (x = y \rightarrow \forall P (Px \equiv Py))$ expresses the indiscernibility of identicals, if x is identical to y, then x has all its properties in common with y. When the standard Peano axioms, and especially the *induction* axiom, are put in second-order logic, they are sufficient so that, for PA2 the conjunction of these axioms, they determine a structure (up to isomorphism) and for any arithmetical statement A, if A is true, then PA2 $\rightarrow A$. Now the the axioms include a constant *successor* function S and object quantifiers assumed to range over *numbers*. But the structural point is that any entities and functional relation for which the axioms hold will do. We might as well talk about dogs arranged by height, so long as the sequence has all the right features. Thus we may rewrite our formula, substituting a function *variable R* for S, and letting the object quantifiers range over entities in an arbitrary class X (that is, $\forall x (\ldots x \ldots)$ becomes $\forall x [Xx \rightarrow (\ldots x \ldots)]$). Then, where the subscripts indicate that we are working with expressions so revised, it remains that, $\forall X \forall R [\text{PA2} \rightarrow A]_{RX}$. This

is the content of the point that *it does not matter* what the objects are and what the relation is — so long as they are arranged in the right way as to satisfy the structural requirements from the axioms. Observe that we are not talking about *structures* as such, but about *objects* which are arranged in the appropriate way. Thus we might have a sequence of Zermelo sets, von Neumann sets, a sequence of dogs, or whatever. All that matters that there are things arranged in the right way.

So far, this is not purely nominalistic. On the standard account, first-order predicate letters are interpreted by sets of individuals. And second-order quantifiers are understood to range over such sets. So far then, the theory requires sets, where the sets themselves are not concrete. But Hellman utilizes an ingenious strategy worked out in the appendix to Lewis, *Parts of Classes*, combining mereology and plural quantification for a way out. Again, I only sketch the basic ideas. First, plural quantification (see Oliver and Smiley, *Plural Logic*; seminal papers are Boolos, "To Be Is To Be a Value of a Variable," and "Nominalist Platonism"). A famous sample sentence is, 'Some critics admire only one another'. Given that the class of critics may be any size, standard representations of this require quantification over classes of critics. In second-order logic, it goes into, $\exists X(\exists u Xu \wedge \forall u[Xu \rightarrow (Cu \wedge \forall v(Auv \rightarrow Xv))])$; there is a non-empty class such that anything in it is a critic, and anything it admires is a member of the same class. But the ordinary claim does not seem to be about classes at all — it is rather about critics. This suggests the essential use of a sort of quantification distinct from that of standard treatments. Thus in addition to $\forall x$ and $\exists y$ we might allow quantifiers $\forall xx$ and $\exists yy$, read *for any things xx*, and *there are some things yy*, along with relations of the sort, $t \prec T$ to say that t is among the T's. Then we might say, $\exists xx \forall u[u \prec xx \rightarrow (Cu \wedge \forall v(Auv \rightarrow v \prec xx))]$; there are some xx such that if u is among the xx then u is a critic and any v it admires is among the xx as well. And *this* does not seem to have anything to do with classes at all. Boolos shows that second-order claims with just unary predicate variables (that is variables that take just one individual term, of the sort Xx, rather than Xxy or $Xxyz$) can be completely duplicated by means of plural quantification — and so that the logic of such claims can be carried out without (apparent) appeal to sets.[21]

This is nice, but mathematics in general, and PA2 in particular, requires more than just the effect of monadic predicate variables achieved by plural quantification. But again there is a way out. The standard interpretation of a relation symbol is a

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[21]The technical claim is secure. But the philosophical one is controversial: Given that we can reduce one thing to another, there is the option of going the other way. So one might suggest that the possibility of duplicating systems of plural quantification in second-order logic, demonstrates its commitment to sets! But observe that the natural interpretation of our original plural claim seems to require just the critics, and so not the classes.

set of ordered individuals. Thus the interpretation of Lxy may be a set $\{\langle$Romeo, Juliette$\rangle$, $\langle$Juliette, Romeo$\rangle$, $\langle$Bill, Hillary$\rangle \ldots\}$. With just the resources of monadic predicate variables, one gets the effect of quantification over such things so long as ordered individuals are themselves members of the domain. For this, Hellman follows Burgess, Hazen, and Lewis (from the appendix to *Parts of Classes*) to a brief excursion into mereology. Again roughly, on such an account, there exist arbitrary sums of individuals. So if Sleepy and Dopey are dwarfs, then something is Sleepy-Dopey, not itself a dwarf, but with Sleepy and Dopey as dwarf parts. And similarly for other combinations of things. For the effect of ordering individuals, Hellman requires that there be at least infinitely many atomic individuals. Thus, put by means of plural quantification,

AI There are some individuals one of which is an atom and each of which, combined with an atom not part of it, is also one of them.

So if an atom a is one of the individuals, then combined with an atom not part of it some ab is one of them; so some abc is one of them; and so forth. So there are infinitely many atoms themselves parts of the individuals. One of the standard points about infinite totalities is that the whole corresponds to the proper parts. So consider some such totality T in correspondence with wholly distinct parts $p1$ and $p2$,

$p1$	a	c	e	g	i	k	m	o
	|	|	|	|	|	|	|	|
T	a	b	c	d	e	f	g	$h \ldots$
	|	|	|	|	|	|	|	|
$p2$	b	d	f	h	j	l	n	p

(but there is no requirement that $p1$ and $p2$ exhaust T). Now the sum bd, say, is unordered. However, given our mapping, the object ch unambiguously associates b with the object $p1 = acegikmo\ldots$ and d with $p2 = bdfhjlnp\ldots$. For c is a part of $p1$ and h is a part of $p2$. Suppose $p1$ identifies the first place of a pair and $p2$ the second. Relative to specification of these objects, then, ch may be treated as representing the ordered pair $\langle b, d \rangle$. And for any x, y in T, there is some u in $p1$ and v in $p2$ such that the sum uv unambiguously represents $\langle x, y \rangle$. In the appendix to *Parts of Classes*, given just the resources of plural quantification, Burgess, Hazen, and Lewis show how to set up such a correspondence, and so such a notion of relative pairs. And given the pairs, we may represent, say $\langle a, b, c \rangle$ by $\langle a, \langle b, c \rangle \rangle$. And similarly for other sequences of individuals.

Now without the quantification implied in second order logic, given just that there are the required individuals, we return to the fundamental structuralist point: All that

matters is that there are things arranged in the right way, be they Zermelo or von Neumann sets or dogs, and the resulting mathematics is about them. Suppose then that our mathematical claims have been adjusted to reflect plural quantification over mereological objects as above, where this modification is reflected again in a modified subscript, $\forall X \forall R[\text{PA2} \to A]_{RX*}$.

Of course, if we are nominalists, there might not be any Zermelo or von Neumann sets, and there are not likely to be enough dogs either. Perhaps there are enough physical things in the universe to do duty for the integers, and perhaps not. This is a point of contention for traditional nominalist programs. Even if there are sufficient things, however, it seems inconsistent with the eternal and necessary character of mathematical truth to make mathematical truth dependent on what is concrete. Even if there are not enough things, however, it seems plausible that there could be enough things. Thus Hellman proposes that the mathematical claims are meant to range over all possible arrangements of things. So with $\Box$ for necessity and $\Diamond$ for possibility, $\Box \forall X \forall R[\text{PA2} \to A]_{RX*}$. Additionally, $\Diamond \exists X \exists R[\text{PA2}]_{RX*}$. So it is no part of the theory that the axioms are satisfied — though it is required that they could be. For details and additional postulates, see Hellman.

This is an interesting and powerful approach which recovers an enormous amount of ordinary mathematics. It seems to recover as much mathematics as is required for science. No small achievement! For now, I make just a couple observations. First, the sum of a, b, c is just abc — as is the sum of a and bc. So by simple sums of things, there is no distinguishing, $\{a, b, c\}$ and $\{a, \{b, c\}\}$ which have the same atoms, but are distinct. That is why it was so much work to identify pairs of things, with just their sums as ground. And given plural quantification with the effect of pairs, we get something like quantification over sets of pairs. But we do not achieve anything like quantification over sets of arbitrary rank (sets of sets of sets. . .). So by the nominalistic means described so far, without abstracta, not all of classical mathematics is recovered. Second, Hellman introduces a modal operator into his account. So long as he retains his nominalism, he cannot account for this notion by means of abstract models, as a classical logician might do. And similarly, he cannot account for it on the basis of abstract possible worlds or the like. Hellman says the modality is primitive. So he trades abstract ontology for modality. But this will not have been much of a bargain, if the primitive is somehow unacceptable.

1.3 Problems about Primitives

At one level, reasons for platonism are themselves objections at least to hard nominalism. And reasons for nominalism are objections at least to hard platonism. So considerations from one methodology count as objections to considerations from the other. This is particularly obvious in the case of our basic reasoning for nominalism, which began in the form of an argument against platonism. But similarly from the other direction as well. It is likely that advocates of one view will think that compulsory questions are left unanswered by the other — the platonist, that nominalism fails to account for commitments of data which cannot be denied; the nominalist, that platonism exceeds the reach of any theory that could plausibly be founded on available data. But problems do not remain at the methodological level. Rather, they reappear as problems about primitives in theories put forward from both sides. We can see this by returning briefly to our examples.

Every theory has its primitives. So it is no knock that a theory has them. Contrast a medieval who explains motion of the planets by their "motive power" with a Newtonian who explains their motion by "gravitation." Perhaps, through his mathematical characterization of gravitation, the Newtonian explains and unifies phenomena which the medieval does not. But the idea of a gravitational force seems, at bottom, no less mysterious than that of a motive power. Each is, in its respective theory, primitive and unexplained. Perhaps gravitational forces are explained in general relativity or some other theory. But these will have primitives of their own. And similarly in philosophy. Thus, in a famous example, when push comes to shove between Quine and McX in their debate on properties from "On What There Is," McX proposes to explain the redness of houses, roses and sunsets by a common (abstract) attribute of redness. Quine says "that the houses and roses and sunsets are all of them red may be taken as ultimate and irreducible, and it may be held that McX is no better off, in point of real explanatory power, for all the occult entities which he posits under such names as 'redness'" (10). Quine is right that the having of primitives is not by itself an objection against one or the other. But theories will count as better or worse insofar as they do or do not explain and unify phenomena which others do not. And theories may themselves identify proper "locations" for their primitives. On this, consider Epicurus's atomic theory on which all the atoms were initially uniformly moving in a single direction; as a result of a "swerve" collisions were introduced into this system, resulting in the world with all its complexity. This is a reasonably sophisticated theory! The atoms, initial motion and swerve are all primitive. But one might think that atoms and initial motion are legitimate in a way that the swerve is not. The point seems *internal* to Epicurus's own theory.

1.3.1 Problems for Platonism

Consider again van Inwagen's point from above (p. 20) that if we set out to describe the intrinsic nature of a pen or the like, we will have a great many things to say — about the nature of the ink, the working of the ball, or whatever. But not so for abstract objects in general, and properties in particular. Thus van Inwagen sets out to describe just a certain role that properties may fill. Problems about intrinsic nature underly also Benacerraf's point about numbers. Thus either of, $\{\phi\}, \{\phi, \{\phi\}\}, \{\phi, \{\phi\}, \{\phi, \{\phi\}\}\}$, or $\{\phi\}, \{\{\phi\}\}, \{\{\{\phi\}\}\}$ seem equally fitted for the role of the first three positive integers. But the situation is worse than this. There are, of course, other sequences of sets that would do the job. Further, in "Straight Talk About Sets," Michael Jubien argues that sets are best thought of as certain properties. Thus, $\{x, y\}$ is the property BEING X OR Y. And Maddy, *Realism in Mathematics* has another non-traditional account. So it is controversial what *sets* are supposed to be. Others would reduce properties to sets. The number role, then, grossly underdetermines the nature of the entities that would fill it. And similarly across the board for abstracta. Thus Wolterstorff and Thomasson differ about the nature of entities in the role of fictional characters — a dispute which van Inwagen avoids "by the clever expedient of being vague."

With considerable trepidation, I think we can illustrate the problem by considering Hillary Putnam's famous "model theoretic" argument (in, for example, *Reason, Truth and History*). At one level, the point of this argument is straightforward. Suppose, as on the Quinean picture, we start out just with certain formal sentences as data. Then, Putnam observes, the truth values of those sentences do not suffice to determine the referents of the terms in them — that is, one may hold truth values of sentences constant, with all sorts of variation in reference. This is a simple result from logic, and may be illustrated as follows.

Suppose a standard interpretation of a formal language on which constant symbols are assigned to particular things, and predicate letters to sets of individuals. In anticipation of a pair of concrete interpretations, consider a simple abstract structure S. S has a universe $U = \{o_1, o_2, o_3, o_4\}$ and assignments as follows,

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\begin{aligned}
S: \quad S(r) &= o_1 \\
S(m) &= o_3 \\
S(D) &= \{o_1, o_2\} \\
S(C) &= \{o_3, o_4\} \\
S(P) &= \{\langle o_1, o_3 \rangle, \langle o_1, o_4 \rangle, \langle o_2, o_3 \rangle, \langle o_2, o_4 \rangle\}
\end{aligned}
$$

We shall see that this can be given some intuitive content. But first, observe that from a structure of this sort the truth value of any sentences involving these predicates and

terms is completely fixed. Thus, for example, $\neg\exists x(Dx \wedge Cx)$ and $\forall x(Dx \rightarrow Pxm)$ are true. In the latter case, it is enough to observe that Pxm and so the conditional is satisfied for o_1 and o_2, and Dx is unsatisfied, so that the conditional is, for o_3 and o_4; so the conditional is satisfied for any member of U. And similarly in the other case. By this reasoning, the sentence will be true on any interpretation structured like S. Or, generalizing and putting the same point another way, *any interpretation with structure S must make all the same sentences true.*

Now consider a pair of concrete interpretations I and J with objects matched to o_1, o_2, o_3, and o_4 as follows,

$$
\begin{array}{ccccc}
I: & \text{Rover} & \text{Fido} & \text{Morris} & \text{Sylvester} \\
 & | & | & | & | \\
 & o_1 & o_2 & o_3 & o_4 \\
 & | & | & | & | \\
J: & \text{Rover} & \text{Morris} & \text{Fido} & \text{Sylvester}
\end{array}
$$

The concrete interpretations are specified in relation to the structure S, and (since order does not matter) have their universes the same — consisting of just Rover, Fido, Morris and Sylvester; but for example, $I(D) = \{\text{Rover, Fido}\}$ and $J(D) = \{\text{Rover, Morris}\}$; and similarly for other cases. I is the *natural* or *intended* interpretation. On this account r is Rover, m is Morris, D ranges over the dogs, and C over the cats; Pxy just in case x pursues y. Then $\neg\exists x(Dx \wedge Cx)$ says that nothing is both a dog and a cat; and $\forall x(Dx \rightarrow Pxm)$ that every dog pursues Morris. Each of these is true on I. It is not so easy to give an intuitive account of interpretation J. But since it has the same structure as I, it makes all the same sentences true as I. But now we have a case where truth values for (all) sentences remain the same, while references for constant symbols and predicate letters do not — so it appears that truth values for the sentences do not suffice to determine references for the predicates.

Observe that what we have done is merely to *permute* or shuffle the objects of one interpretation relative to another. Thus, given objects as for interpretation I, we have a *permutation function p* which links each object on I to a "mate" on J; thus we have $p(\text{Rover}) = \text{Rover}$, $p(\text{Fido}) = \text{Morris}$, $p(\text{Morris}) = (\text{Fido})$, and $p(\text{Sylvester}) = \text{Sylvester}$. Then interpretation J can be seen as like I except that for each object x in the interpretation of a predicate or term, instead of x, we substitute $p(x)$ in its place. Interpretations which are related in this way are said to be *isomorphic* — and it is a standard theorem from intermediate logic that isomorphic interpretations make all the same sentences true. And this is because, as above, isomorphic interpretations share a common structure and interpretations with the same structure make all the same sentences true. So this is Putnam's strategy: he says, in effect, "Suppose some

specification of truth values for sentences — as many sentences as you like; given any account of reference for these sentences, consider a permutation *p* on objects of the domain (indeed, there will be many such permutations), and the corresponding isomorphic interpretations; but these interpretations have all the same sentences true with different references for the predicates. And if you want to fix truth values at different possible worlds, I can do the same trick at as many possible worlds as you like, so as to alter reference at them all. It follows that truth values do not determine reference."[22]

Now this argument has excited enormous controversy. In some sense, Putnam aims to undercut ordinary realism with respect to concrete physical objects. I do not for a moment think these arguments succeed. But it is worth noting that standard replies grant that something beyond truth values of sentences we accept is required to fix reference for predicates and individual constants. Thus a typical response is that reference depends on external connections to things, and in particular that reference requires some specifically *causal* connections with the things to which we refer. So the idea is that reference is fixed not merely by truth values of sentences, but also by these (at least partly causal) external relations between words and things (see, for example, Devitt, *Realism and Truth*). Alternatively, one might appeal to intrinsic "joints" in the world itself that would separate interpretations that are intended from ones that are not (see, for example, Lewis, "Putnam's Paradox").

But starting with just the Quinean data of sentences whose truth values we want to preserve, these seem to be what the platonist cannot provide. If there is ever a context in which Putnam's argument would seem to get some purchase, it is against a Quine-style argument for abstracta. The point here is not merely that causal isolation brings with it problems about reference — a point mentioned already by Benacerraf — so there might be abstracta and we only have problems talking about particular ones. But it is that data from truth values alone do not give sufficient content to identify determinate abstract things at all. Or, alternatively, from this basis, supposing it is so much as possible to offer theories of particular abstract things, such theories must be no more than base speculation insofar as the criteria we are able to lay down might be satisfied in utterly arbitrary ways. From platonism, there are things which must have some natures, but the top-down approach leaves us with nothing but roles that must be filled. As van Inwagen says, the resulting theories are nearly vacuous — and insofar as we try to say more, there seems to be no way of adjudicating the issue.

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[22]This argument, made in the appendix to *Reason, Truth and History* in terms of permutation and isomorphism, is sometimes made by means of the Löwenheim-Skolem Theorem — another result from formal logic according to which different interpretations may make all the same sentences true (in this case, interpretations with different size universes). See, for example, Putnam, "Models and Reality."

This is not merely the nominalist's original point, that theories of abstract objects outrun their evidential base. Rather, it is to observe that the platonist's own data does nothing to illuminate the nature of abstract things apart from extra links "from below" — resources of the sort the nominalist seemed to require all along. So far, then, Van Inwagen seems to have it right: the platonist can talk about roles filled, but not about that which fills the roles.

1.3.2 Problems for Nominalism

Similarly positive nominalist theories may be pushed in the direction of that which the platonist seems to require. For the case of modal fictionalism and possible worlds, it will be instructive to consider some complaints urged against alternate approaches. In *On The Plurality of Worlds*, Lewis develops his account on which worlds are things like the universe in which we live, though different ones are spatiotemporally and causally isolated. A thing is possible iff it is so in some world, necessary iff it is so in all. But one might object that there are no such things, and attempt to attain the benefits of this paradise on the cheap. Thus Lewis considers certain abstract entities that would, from his point of view, count as substitute (ersatz) worlds. One particularly straightforward proposal is *linguistic* ersatzism. On this account, worlds are some collections of sentences; an ordinary sentence is true at a world just in case it is a member of the world; then a thing is possible iff it is true at some world, and necessary iff it is true at all. Supposing that there are sets of sentences, the question becomes whether these entities are sufficient for an account of modality.

An ersatz world substitutes for a universe, with all its myriads of detail. This puts burdens on the expressive power of the language — for the language should be sufficient to describe anything about a world. And, similarly, philosophers have been led to say ersatz worlds are *maximal*, that for any sentence *a*, either *a* or not-*a* is a member. In chapter 3 we shall encounter difficulties related to each of these points. For now, suppose that some entities manage to describe ways the world could be.

Even so, Lewis objects that ersatz worlds cannot be specified apart from appeal to modality. One response is that collections of sentences count as possible worlds when they are consistent relative to some axioms and criteria for entailment. So if axioms include, "no dog is a cat" and a story says Fido is both a dog and a cat, contradiction follows in the usual way, and the story is not consistent. So far, so good. But, says Lewis, any plausible specification of the axioms will itself rely on primitive modality.

> Primitive modality will not go away. The axioms to do the job may exist,
> but the ersatzer will not be in a position to specify them. He can only
> declare: the axioms shall include whichever sentences of such-and-such

> form are necessarily true. Once he says that, all his analyses from there
> on are modal. (154)

This may be correct to the extent that you or I cannot do the job except as Lewis says. Still, here is a sort of "brazen" reply: Suppose some sets are all the worlds, or some sentences are a specification of all the axioms. Seemingly god, at least, could survey the sets there are, "point out" the relevant ones, and say, "thus I identify the possibilities." So god could give an analysis of modality in terms of linguistic entities making no appeal to modality. But for an adequate analysis of some feature of the world, it should not matter who does the analysis. Lewis says worlds are determinative of modality; the ersatzer, god in this case, says that particular sets and sentences do the job. Each "points" and says, "*voilà*." And relative to god, the human ersatzer's problems may seem to be (merely) epistemic and practical. Lewis grants that the ersatzer may legitimately depend on knowledge of modal facts for axiom specification. But it is unlikely that some modal facts will ever be known. So the human ersatzer is in the position of being unable to determine whether certain sentences are axiomatic or not. And perhaps there are "too many" axioms for any finite being to specify them all. But, again, these are not problems for ersatzism as such. So far as primitive modality goes, Lewis and the ersatzer are in very much the same boat. When they "point," it is required that they point to all and only the possibilities. In either case, once this is done the analysis is complete without appeal to modality.

Now there is something unsatisfying about this brazen reply to Lewis. But I think the worry points to a deeper problem with application to views of *both* sorts — deriving from their "primitive" claims. For any entities identified as "worlds," one wants to know why those entities are the ones that are determinative of modal truth. In fact, it appears that relevant primitives are mislocated. Suppose, for example, that as an analysis of the modal truths someone were to point at a stack of books and assert that, for any sentence P, possibly P just in case P appears in some book in the stack. Perhaps the claim is extensionally correct. Still, you might respond that the claim is bizarre — there is no appropriate connection between the sentences in the books and the modal facts. Many modal truths seem to be independent of what is in the books, and plausibly the modal truths are determined by something else. Thus even if it actually happens that possibly P iff P is in a book in the stack, if the modal facts are independent of the books, the proposal cannot count as an analysis of modality. For any entities identified as the possible worlds, we may wonder what privileges *these* entities as ones that are relevant to modal truth.

From the start, one may think that modal *fictionalism* would run into van Inwagen's concerns about fictional entities. But assume Rosen is safe here. How does

his view fare fare against objections from above? We have seen a sort of tradeoff between packing content into specification of fictions, and primitive modality for their specification. To the extent that fictions do not include everything there is to say about the universe of worlds, modal fictionalism requires primitive modality. Against this, the ersatz program mounts a defense by building complete descriptions into the stories, and letting the ersatzer point for specification. So far, then, the stories are specified without appeal to modality. But Rosen seems to have no such option available — or, rather, insofar as such an option is available, his view collapses in the direction of something like linguistic ersatzism, on which the fictions turn out to be like the ersatzer's abstract worlds. But then the view is not nominalistic at all.

Again, even if the view is extensionally correct, like a stack of books, fictions do not seem apt for an analysis of modal truth. Thus we reached the conclusion that "there is something bizarre in the thought that modal statements are true in virtue of the content of some fiction which might not have existed, and didn't exist a long time ago" (p. 28). So the relevance worry extends from Lewis's view and linguistic ersatzism to Rosen's fictionalism. Certainly *pointing* seems to put the primitive in the wrong place. Rosen might pack all of modality into the primitive. Then there is no problem of relevance, but also it is not clear what work is left for the fictions. In chapter 3 we shall offer an *account* to explain how modal claims may be a quantification over some entities. But so far, we we have seen no such account.

Similarly, modal mathematical structuralism may also seem to press on its primitives in a way that is problematic for nominalism. So Quine, in "Reply to Charles Parsons" from the Hahn and Schlipp volume says,

> Goodman and I got what we could in the way of mathematics, or more directly metamathematics, on the basis of a nominalist ontology and without assuming an infinite universe. We could not get enough to satisfy us. But we would not for a moment have considered enlisting the aid of modalities. The cure would in our view have been far worse than the disease. (397)

If modality requires that which cannot be accounted for on nominalistc grounds, then modal structuralism simply is not a nominalistic account. Again, then, the positive nominalistic theory may sit uneasily with a method that is supposed to restrict quantification to actual concrete entities.

In either the platonistic or the nominalistic case, then, we encounter difficulties about locating, or at least working out our primitives in the place where they belong, given the momentum of the very methods with which we begin.

1.4   Towards Reconciliation

In this final section, I say just a bit about how grounded platonism is supposed to provide a solution to problems from platonism and nominalism, and how I propose to get there.

1.4.1   Toward a Solution

Supposing it can be made out, grounded platonism combines soft versions of both platonism and nominalism. So there are abstract objects, and all objects are grounded in, and so dependent upon the concrete. Consistent with Lewis's doctrine of Humean Supervenience, there is a distribution of concrete qualities, and the abstract universe supervenes on that. In contrast with the views of the typical Humean, however, among things that supervene, are ones that are abstract. We accept the burden of explaining the supervenience or grounding relation. And, supposing that this can be done, there is a basis for a response to concerns from both platonism and nominalism.

On the one hand, the Quinean arguments do not by themselves require the hard version of platonism. The requirement is rather that existential claims be satisfied. We get hard platonism, presumably, only insofar as it seems that there is no path "up" from concrete things, to ones required to make the existential claims true. But, if there is such a path, the ground is pulled out from under this concern. And given an account of the relation by which abstract objects are grounded in the concrete, there is room for a response to other objections as well. Consider again the case where ordinary physical objects do not enter into causal interactions at all, but supervene by a relation of *constitution* on ones that do (p. 23). The force of epistemic arguments against abstracta does not require specifically causal connections to concreta, so much as some explicable connections. And if we are able to explicate the supervenience relation, then there is room for knowledge of abstracta, no less than there is of ordinary objects, on the supposed picture. And, similarly, problems about the intrinsic nature of abstracta get off the ground insofar as there are not the sorts of connections to abstracta that there are to concreta. But, again, if we are able to explicate the supervenience relation between abstract and concrete things, then there is room for a story about their determinate natures, again no less than there is for ordinary objects on the supposed picture about concrete things.

Similarly, the bottom-up methodology does not by itself require the hard version of nominalism. The requirement is rather that requisite connections exist. Again, consider the case where ordinary physical objects do not enter into causal interactions, but supervene by constitution. Just given the lack of causal interactions, the nominalist

need not *deny* that the ordinary objects exist. Supposing, then, that connections along the same lines can be made out for abstracta, grounded platonism is in a position to satisfy nominalistic methodological considerations. And, at the same time, it makes room for entities of the sort that Quinean method seems to require. Thus, for example, it remains to be seen whether fictional entities, properties, numbers and the like are among the entities that supervene on the concrete base. If there are such things, then pressure to distort theories in order to restrict quantification is removed — so there is room for the pleasures of platonism, without the pain.

1.4.2 A Preview

In this chapter, I have tried to explain something of what is at stake for grounded platonism. It remains to work out the view. Chapter 2 develops accounts of *proper* and then *ordinary* things. Discussion centers especially on the case of a statue which cannot be smashed into a ball, though the clay which makes it up can be so smashed. I survey accounts on which the lump and statue are co-located but distinct proper objects each with their own modal properties, on which one is a proper object and the other is not, and on which there is a single proper object but with no modal properties at all. I find all such views wanting. I defend an approach on which the proper things do not include portions of clay and statues of the sort that can or cannot be smashed into balls. Things proper do have modal properties, but these are not of the sort usually proposed. Rather, modal properties of these things depend on *kind* properties as BEING SOME CLAY or BEING A STATUE, and the modal properties are of the sort, BEING SUCH THAT POSSIBLY SOMETHING IS THE SAME CLAY AS THIS AND IS SMASHED INTO A BALL and BEING SUCH THAT NOT POSSIBLY SOMETHING IS THE SAME STATUE AS THIS AND IS SMASHED INTO A BALL.

But once we offer an account of this sort, the door is open again for an account of coincident objects. These will not be things proper. Rather, kinds are shifted into the account of *ordinary* things. Ordinary things turn out to include lumps, statues, rocks, tables, chairs, and us. An ordinary thing is constituted by some objects, ordinary or otherwise, along with a kind which provides a sort of function or map from constituting objects to the resultant ordinary thing with all its properties. Thus an ordinary thing results from constitutive objects and a kind. The account is general enough to allow objects of different types to count as ordinary, including, say, events, and objects that are abstract as well. Thus, fictional objects and sets, for example, may be perfectly ordinary things, grounded on a concrete base.

Insofar as it apparently outruns the resources of actuality, modality is a particularly pressing problem for grounded platonism. Already chapter 2 is partly driven by modal

considerations. In chapter 3, I take up modal concerns more directly. The chapter begins with discussion of possible worlds and especially *linguistic ersatzism* as a theory of worlds. After some preliminary setup, I take up objections with respect to the metaphysics, and then the relevance of such worlds. On the first count are objections according to which no linguistic entities are adequate to represent all the facts of modality. In particular, I consider objections according to which actual languages are not adequate to represent non-actual and qualitiatively indiscernible individuals, and according to which, supposing an adequate language, no sets are *maximal* in the sense that for any $\mathcal{P}$, either $\mathcal{P}$ or $\neg\mathcal{P}$ is a member. On my account, an ersatz theory survives, but in considerably modified form. It is also possible to argue that worlds are irrelevant to modal truth: Either worlds are constrained relative to actuality or not; if they are not, worlds may seem irrelevant to modal truths which are themselves constrained relative to actuality; and if worlds are so constrained, they may appear as superfluous if the modal facts are prior to the worlds. I argue that worlds do have a sort of relevance to modal truth, but only relative to the actual intrinsic natures of non-modal properties. It is the burden of much of the chapter to explain this relation. The resultant analysis of modality is reductive in the sense that modal properties are explained in terms of properties which, taken individually are not modal. The chapter concludes with some applications of the view, including to a simple version of the modal ontological argument.

chapter 4. . .

chapter 5. . .

The discussion is "upside down" in the sense that earlier chapters may depend on ones that follow. So, in discussion of ordinary things, I appeal both to possibilities and properties. And the account of possibilities appeals not only to the prior discussion of ordinary things, but to certain features of the properties. For each part, however, it should be enough that there is some account of the relevant subject matter from the others — not necessarily the one developed here. To this extent, the chapters are independent, though of course, the collection makes the cumulative case for grounded platonism.

Each of the core chapters 2, 3 and 4 includes a section "towards a formal model." These parts are meant to demonstrate coherence and flesh out certain details. They include also some applications and extensions of the view. But I do not offer anything approaching a formal "theory of the world." And, particularly if you do not swim easily in formal waters, they might be read lightly on a first time through, for just the general idea.

Strangely perhaps, I do not see the project as speculating or postulating about the furniture of the universe — at least in the usual sense. We do not notice a theoretical

need and postulate some range of objects: essences, numbers, or whatever to fill that need. Rather, beginning with the claim that there are ordinary objects on the order of tables or chairs, we observe the rich array of things the world has to offer in addition to these. This puts the philosopher who sets out to explain issues in fiction or mathematics in the position of a scientist who, having noticed some phenomenon, sets out to explain it in terms of fundamental entities already understood. Of course, as in the physical case, the account as a whole receives support insofar as it is successful in explanation of the cases. But it is important that we do not have merely a "top-down" motivation of the sort associated with speculative platonism. Rather, there is also the "bottom-up" start in the distribution of qualities, and structure of things built on them.

Chapter 2

Things, Ordinary and Otherwise

As before, begin with a distribution of qualities. From AS concrete objects are ones with some particular qualities as constituents. And grant that all objects have a ground in ones that are concrete (SN). Then for grounded platonism (GP), we require the result that among things so grounded, some are objectively existing abstract objects (SP). This chapter reaches the result that some objects are abstract. On this account abstract objects are *ordinary* things, of a kind with rocks, tables and organisms.

The chapter begins with sections on the problem of modality and critical discussion of some different approaches to things. I then turn to positive accounts of *proper* and *ordinary* things. Discussion centers especially on the case of a statue which cannot be smashed into a ball, though the clay which makes it up can be so smashed. I defend an approach on which things *proper* simply do not include portions of clay and statues of the sort that can or cannot be smashed into balls. Things proper do have modal properties, but these are not of the sort usually proposed. Rather, modal properties of these things depend on *kind* properties as BEING SOME CLAY or BEING A STATUE, and the modal properties are of the sort, BEING SUCH THAT POSSIBLY SOMETHING IS THE SAME CLAY AS THIS AND IS SMASHED INTO A BALL and BEING SUCH THAT NOT POSSIBLY SOMETHING IS THE SAME STATUE AS THIS AND IS SMASHED INTO A BALL. Such properties might account for things we are inclined to say about the statue and the clay. Granting, however, that no thing proper is an ordinary statue or portion of clay, this account opens the door to an account on which kinds are shifted into the account of *ordinary* things. The ordinary things include statues and portions of clay. But the ordinary things are structured so as to include events, fictional objects, and the like as well. So some ordinary things are abstract.

2.1   The Problem of Modality

The case of the statue and the clay is a problem about modal features of the statue and the clay. This locates it among more general problems about possibility and necessity. Thus this section develops some general perspective about the problem of modality. There is a basic problem about grounds for possibility and necessity, both *de dicto* and *de re*, and a more specialized concern about the *de re* case. For the purposes of this chapter, I presume a solution to the *de dicto* problem along the lines of one that will be developed in chapter 3, and concentrate on whether this puts us in a position to say something about the *de re* case. In this section, I merely develop the problem, beginning with the general case, and then more specifically the problem about *de re* modality and so the the statue and clay.

2.1.1   The Basic Problem

By way of introduction, observe that possibility and necessity come in different varieties. Thus, in one sense, a bishop must move along a diagonal — for to do otherwise breaks the rules. But in another sense, the bishop can move from any square to one of a different color — for nothing is simpler than to pick it up and set it down wherever you please. And similarly different notions of possibility and necessity appear in a wide variety of contexts. (I sometimes ask students if it is possible to drive the sixty miles from our campus in San Bernardino to Los Angeles in thirty minutes. From natural assumptions about Los Angeles traffic, law enforcement, and the like, most say it is not. But some, under different assumptions, allow that it can be done!) It is traditional to focus on a broad *metaphysical* sort of modality. In this sense, it is not possible for something to be round and not round, or for a thing to be round and square; but it is possible for Obama to play in the NBA, and for aliens to invade the earth. More controversially, water is necessarily H_2O; and it is possible for natural laws to be other than they are, and so, say, for a thing to travel faster than the speed of light. I shall concentrate on this modality, though the structure of the problem is parallel from one case to the next. Also, in one variety or another, modal notions are ubiquitous. Fundamental components of logic depend on possibility and necessity. Even if philosophical results are not necessary, aspects of philosophical reasoning remain subject to necessary constraint. One encounters modal notions in science, for natural law, and in moral or decision theory, where it matters what one can and cannot do. And so forth. It is natural, then, to seek an adequate account of these notions. But it is not obvious that any such account is to be had.

Problems about possibility and necessity as such are parallel to traditional prob-

lems about knowledge of possibility and necessity. In each case, on a plausible account, supposed results outrun their grounds. So it may help to begin with a sketch about worries for knowledge. It is natural to think that knowledge of the world is derived from observation of the world. Suppose we observe that all crows are black. Is it possible that there be a non-black crow? The answer is not determined by our observation that all crows are black, for not every possible situation is actual. If it is possible for there to be a non-black crow, then some non-actual possibility is such that *if* it were actual there would be a non-black crow. But we observe only the actual world; there is no observation of what is non-actual. And if there is not any observation of what is not actual, then there is not any any observation of correlations between the actual and the non-actual — it is not obvious how observations of the actual are relevant to what is merely possible. Thus, following a suggestion from McGinn ("Modal Reality," 177-182) consider a theory *T* with consequences for both non-modal properties of the actual world and non-actual possibilities, and some other theories whose actual-world consequences are the same as *T*, but non-actual consequences are different. Insofar as observation is of actuality, there is seemingly no observation to distinguish among the theories. So the empiricist, who grounds knowledge of the world in observation, has reason to hold that observation does not ground knowledge of truth or falsity for the various modal claims, and may find in these considerations the basis for a general skepticism about modal knowledge.

This skepticism rests on a metaphysical difficulty according to which the requirements for possibility and necessity exceed the resources of actuality. The basic elements of the problem lie not very far under the surface of the possible worlds picture of modality. Thus consider, for example, Lewis's modal realism (as, *On The Plurality of Worlds*). As we have seen, on Lewis's view, for every way a world can be, some world is. A thing is necessary if it is true in all worlds, possible if true in some. On this view, worlds are things very much like the universe in which we live, though different worlds are spatiotemporally and causally isolated; for Lewis, 'actual' is analyzed indexically — the reason this world is actual for us is that it is the one we are in. Say a property is *categorical* iff it does not depend on properties at other worlds. So a thing's BEING ROUND or BEING 10 KILOGRAMS is categorical, where its BEING NECESSARILY ROUND, or BEING POSSIBLY OTHER THAN 10 KILOGRAMS are not; for the former, but not the latter, do not depend on how things are in other worlds. On Lewis's view, there is a plurality of worlds, where each has a distribution of its own categorical properties. In any world, every property, modal or otherwise, has a ground in the overall distribution of categorical properties. Modal properties in a world are not grounded merely in the categorical properties of their own world, but in the categorical properties of the plurality as a whole. Thus crows are possibly non-black in a world

just in case there is some member of the plurality in which there is a non-black crow; crows are necessarily black just in case they are black in every world.

So on this worlds picture, modal properties are not wholly grounded in actuality. But this stands in tension with the natural thought that the very idea of a(n actual) property or truth not wholly grounded in actuality is bizarre or occult. Adapting a case from Jubien ("Problems With Possible Worlds"), suppose god annihilates worlds where I am in another line of work. My natural reaction would not be, "So much the worse for it being possible that I am in another line of work," but "So much the worse for the relevance of possible worlds." Somehow, the way things are in this world makes it possible for me to be in another line of work. Or take a salt tablet. It is natural to say that it is soluble, and that it is soluble because of the categorical way it actually is. Diamonds, plastics, and the like are insoluble because they are categorically different. And similarly for a wide variety of properties. Suppose, for example, I claim to have "ultimate greatness" and insist that my dopplegänger in a world categorically the same as ours does not have it (or has it on alternate Thursdays); when pressed for those features in virtue of which I have it and he does not, I appeal to no categorical feature of the world: my response is just, "It is a brute fact — there is no difference beyond our differing with respect to ultimate greatness." This is bizarre. Similarly for serious moral properties. And similarly for the modal properties with which we began.

Underlying these examples is a perspective about what it is to *have* a property: A thing has one property rather than another when there is some modification of the thing; and the categorical properties are set up so that modification of a thing is by its categorical properties — thus it makes no *sense* to say a thing has a property apart from an actual categorical ground. Of course, one might hold that the world is different *modally* when things have some modal properties rather than others. But, as we have already observed, there may be theoretical relations among the primitives of a theory. So Epicurus says there are atoms and the void. Say atoms, initial motion and the swerve are all primitives of his view. But, given the other primitives, it is hard to see how there is room for the swerve (recall p. 35). Similarly we have the suggestion, ultimately to be defended by a positive theory, that a modal primitive is mis-located in a theory of the world. The natural location for a ground is in the categorical properties of the world.

To the extent that such considerations are plausible, we have a full-blown problem about modal truth. For we seem to accept,

MP Things actually have objective modal properties.

AC Any property has an actual categorical ground.

NG Actual categorical properties are not an adequate ground for modal properties.

MP is from the ubiquity of modality. AC is the natural claim about actual grounds. And NG is illustrated by the worlds picture. But these are inconsistent. From MP and AC, some modal properties have an actual categorical ground. And by NG, no modal property has such a ground. Standard responses take the form of denying MP, that things actually have objective modal properties, as in some versions of empiricism; denying AC, that modal properties have an actual or a categorical ground, as in appeals to possible worlds or ungrounded primitive modality; or denying NG and arguing for the sufficiency of some actual categorical ground. The history of philosophy suggests that it is no easy task to make sense of the world without modal properties. We have suggested AC is at least initially plausible and more so given a theory on which modal properties have a categorical ground. So my strategy is to accept MP and AC and argue for the sufficiency of some actual categorical ground.[1]

2.1.2 *De Re* Modality

Traditionally, philosophers have distinguished between modality *de dicto* and modality *de re* — where "possibly there is a blue horse" is *de dicto* and "Obama is possibly a rock" is *de re*. There are different ways to characterize this difference. Intuitively, *de re* modality is supposed to depend on the modal properties of things (or on modes of their properties), and *de dicto* modality on features of a proposition or saying. On a worlds picture, *de re* modality "tracks" particular objects across worlds, where *de dicto* modality does not. So, for example, "Obama is possibly a rock" depends on how Obama is in different worlds; it is true just in case there is a world where he is a rock. In contrast, "possibly there is a blue horse" depends on horses in the different worlds, but without respect to how any given horse is from one world to the next; it is true iff there is a world where one of the horses is blue. On the usual scheme, this distinction corresponds to a formal one according to which a sentence or formula (is *de re* and) expresses a proposition that is *de re* iff it has a subformula with a proper name or free variable inside the scope of a modal operator; and a sentence (is *de dicto* and) expresses a proposition that is *dicto* iff it is not *de re*. On this basis 'Obama is possibly a rock' and 'There is a man such that he is possibly a rock' with their natural

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[1]Such an account, though attractive on its own terms, may also leave room for a straightforward explanation of modal knowledge. Thus the epistemological skeptic might accept, as an analog to AC that observation is appropriate only to the detection of actual categorical properties (whether or not there are primitive modal qualities or non-actual worlds) and so, with NG, conclude that observation does not ground modal knowledge. (Compare, Hume, *An Enquiry Concerning Human understanding*, §7). If modal properties have an actual categorical ground, this reasoning, at least, collapses.

symbolizations, $\Diamond Ro$ and $\exists x(Mx \wedge \Diamond Rx)$ are *de re*. 'Possibly there is a blue horse' with its natural symbolization, $\Diamond \exists x(Bx \wedge Hx)$ is *de dicto*. Let us begin with the assumption that these characterizations capture the same distinction, and focus on the *de re*.

Though he is not enthusiastic about necessity and possibility on any account, it is well-known that Quine, and many others, have thought that *de re* modality is particularly problematic (see, for example, Quine, "Reference and Modality," and "Three Grades of Modal Involvement"). Suppose property entailments or the like underwrite *de dicto* modal principles of the sort, "necessarily horses are animals" and "necessarily no horses are xylophones." Still, these entailments do not obviously suffice for the *de re* case. Given *de dicto* principles sufficient for the result that there is no world where something is both a horse and a xylophone, we do not yet have that something that is a horse in one world is not a xylophone in another. So we do not yet have that Seabiscut, say, is not possibly a xylophone, and similarly, that Obama is not possibly a rock. Insofar as the entailments are necessary *de dicto*, they have no consequences for modal properties of particular individuals. So a solution to the problem of *de dicto* modality leaves the problem of *de re* modality intact.

But the difficulty is not merely that a solution to one leaves the other intact. Rather, Quine argues that there are special difficulties for the *de re* case. By the indiscernibility of identicals, if x is identical to y, then x has the same properties as y. Quine thinks this principle fails for supposed modal properties, and therefore that the very idea of a modal property is incoherent. Quine's own examples tend to depend on definite descriptions, and there are well-known replies. Here is a case, like Quine's, that illustrates the difficulty: 'Necessarily the inventor of bifocals invents bifocals' seems true, and 'Necessarily the first postmaster general invents bifocals', false. But the inventor of bifocals is the first postmaster general. At one level, then, Quine would like to see us as first granting and then withholding BEING NECESSARILY THE INVENTOR OF BIFOCALS to the same individual and so as violating the principle. Arguably, however, these claims are not *de re* at all. Depending on scope considerations, it is enough that no world has something that is and is not the inventor of bifocals, but some world has a thing that is Postmaster General and not the inventor of bifocals. So there is no question about Franklin in different worlds. Against Quine, then, Marcus, Kripke, and others have responded that it is important to separate proper names which may be *rigid* in the sense that they do track with particular things across worlds, from definite descriptions which need not work this way; they suggest that if one does keep them and their roles distinct, problems evaporate (as Kripke, *Naming and Necessity*, 6-15; Marcus, "Modalities and Intensional Languages"; Smullyan, "Modality and Description").

Here is a case that seems to avoid such replies (based on Gibbard's much-discussed, "Contingent Identity"). Suppose god creates *ex-nihilo* a clay statue, and later annihilates it into nothing — with the effect that the statue and the lump or mass of clay of which it is composed coincide over their entire career. On some occasion, a person points to the statue and says, "Let this statue be called '*s*'." *Prima facie*, '*s*' is as good a proper name as any. Similarly, on some occasion, a person points to the clay and says, "Let this clay be called '*c*'." *Prima facie*, '*c*' is as good a proper name as '*s*'. The statue and the clay have their actual categorical features: shape, weight, spatiotemporal location, and so forth, all in common. (And this is so whether we adopt a three- or four-dimensional account of things.) Thus one might think that $s = c$. But then there is a problem if we admit both of the (apparently) *de re* claims,

FS Necessarily *s* is not as flat as a pancake.

FC Possibly *c* is as flat as a pancake.

for then we seem to allow that *c* has BEING POSSIBLY AS FLAT AS A PANCAKE, but *s* does not. With $s = c$, then, there is conflict with the indiscernibility of identicals. But even if the statue is distinct from the clay, insofar as the statue and the clay are categorically the same but modally different, there is a difficulty about AC, the principle that any property has an actual categorical ground. Where both designators are plausibly rigid, it is not obvious how the standard anti-Quinean strategies apply. So it is important for this case that '*s*' and '*c*' are rigid, one no less than the other (see Della Rocca, "Essentialists and Essentialism").

So we began with a general problem about modality. Perhaps this has a solution for the *de dicto* case. This solution does not obviously extend to the *de re* case. But a solution for *de re* modality of any sort may seem hopeless: For we expect modal properties of the same thing — or at least on the same categorical ground, to be the same. But if we grant claims as FS and FC above, they are not the same. Our different approaches to thinghood set up different responses to this case.

2.2 Four Approaches to Thinghood

I begin this section with some notions that set up a general framework for our approaches to thinghood. I then turn to four approaches to things. In each case I find the view wanting. This motivates the accounts of proper and ordinary things in the sections that follow.

2.2.1 The Framework

The case of the statue and the clay is tangled — often with issues not essential to the case as such. Especially, discussion may seem not even to get off the ground depending on idiosyncratic starting points or theses about things. I attempt to present the case in a reasonably neutral way, and especially to bring out the significance of essentialism for solutions to the problem.

Essentialism. Suppose we have in-hand some viable account of *de dicto* modality. Then it is natural to build a response to the problem of *de re* modality on the already existing *de dicto* account. Say we are worried about whether Obama can be a rock. As above, purely *de dicto* modal principles do not give the result that he cannot be a rock. But the problem would go away with the addition of *de re* principles including something like, $\Box\forall x(x$ is-obama $\leftrightarrow x = o)$; then if $\Box\forall x(x$ is-obama $\rightarrow \neg(x$ is a rock)) results from already accepted *de dicto* principles, we get the result that $\Box\neg(o$ is a rock), and so that it is not possible for Obama to be a rock. If there is some property such that necessarily a thing has it iff it is Obama (say, BEING OBAMA or BEING THAT PERSON), then the *de dicto* solution takes us the rest of the way.

This case suggests a schema for what I will call an "essentialist" solution to the problem of *de re* modality. The idea is that things have "linked" to them some properties already associated with the *de dicto* solution (*essences* if you will), and that modal constraints are precisely the constraints already associated with those properties. In the case above, we are given an essence E such that necessarily, a thing has E just in case it is identical to object a, so that $\Box\forall x(Ex \leftrightarrow x = a)$. Then with the *de dicto*, $\Box\forall x(Ex \rightarrow Px)$, the *de re* $\Box Pa$ immediately follows. This is a simple account, and the *de re* principles may appear in somewhat different forms. At any rate, the essentialist strategy is to move from some *de re* principles about essence, through the *de dicto* solution, to the full range of *de re* modality.[2] The strategy does not itself solve the problem of *de re* modality; rather, it only locates it. The full range of *de re* modality is explained with the *de dicto* solution and some "basic" *de re* principles about essence. Assuming a *de dicto* solution, the issue centers on the ground for the basic principles, and therefore on the link between essences and things. As we have

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[2]The *de re* equivalences are subject to niceties about contexts where a does not exist. But these need not concern us just yet. For now, restrict attention to situations where a exists, or assume that the same individuals exist at every situation. (Or, if you like, adopt some other solution to the "problem of variable domains" and make any necessary adjustments to the above.) That *de re* modality requires something like the essentialist strategy seems implicit in Quine's charges of "Aristotelian essentialism" ("Three Grades of Modal Involvement," 175-176, "Reference and Modality," 155-158). For particular instances of the strategy, see notes attached to the different accounts of things.

seen, it is hardly obvious how these principles can have a ground, and so just how the problem can have a solution.

Of course, Obama could hardly lack a property required to be Obama, and the essentialist might argue that essences figure in what it is to be a thing, or in a thing's specification, so as to justify the required principles. In order to flesh this out, and to move in the direction of a positive account, I turn to the particular accounts of what it is to be a thing.

Thing-Candidates. I sketch a range of thing theories and select among them. Some approaches to things allow that there are things which others do not. In setting up relatively "dense" theories, it is thus difficult to avoid what advocates of "sparse" ones will see as much ado about (what is literally) no thing. Thus for example van Inwagen allows that there are simples and organisms, but denies that, strictly speaking, any things are tables and chairs: He says claims according to which "tables are not real . . . darken counsel"; such claims are "perfectly meaningless," just because there are no such things to talk about (*Material Beings*, 99). Of course, others treat tables, or the material of which they are composed, as paradigm instances of objects. But even van Inwagen understands the claim that a former Governor of California has massive biceps (say this is so if his biceps are over ten kilograms). Given that biceps are neither simples nor organisms, this is no relation between Arnold and biceps which are his parts. Rather, it may be that Arnold has the property HAVING MASSIVE BICEPS, or that he stands in a relation to some simples arranged in a particular way. More generally, it should make sense to say of things that they have such-and-such properties at such-and-such spacetime points. Thus van Inwagen and those with related starting points should be able to *understand* — in the sense of being able to paraphrase away — language according to which there are things at those points with various properties.

I work this out in terms of "thing-candidates" — where such talk should be understandable, and so acceptable, from different starting perspectives. Restrict attention to substantial physical things and assume some initial theory about them. Then, first,

PC If a thing has some categorical properties at a spacetime point in a world, there is a *point-candidate* in that world with just those properties.

Where location is itself categorical, point-candidates are identical iff their categorical properties are the same. Distinct things overlapping at a point where they agree categorically result in a single point-candidate there. But it is not required that overlapping things agree in this way: if things overlap where they are categorically

different, there are different point-candidates there (maybe things are fields and fields so overlap). Then,

BC For any set of point-candidates in a given world, the members of that set make a (*world-*)*bound-candidate*.

Bound-candidates have categorical properties depending on the categorical properties of their points; bound-candidates in a world are distinct iff they differ categorically — differing in categorical properties at at least one point. Finally, still ignoring questions about worlds where a thing does not exist (see n2), let an *heirline h* be a function which assigns to each possible world some bound-candidate at that world.[3] Then,

TC For any heirline *h*, there is a *thing-candidate* whose categorical properties at world *w* are just those of *h(w)*.

Thing-candidates have properties, including modal properties, depending on their categorical properties in different worlds. Thing-candidates are distinct iff there is some world where their categorical properties are not the same.

There are a *lot* of thing-candidates. Some are incredibly small, and some are incredibly brief. Some correspond to persons, trees and buildings. But others are instantaneous, point-sized, or scattered across time, space and worlds in unintuitive ways. Insofar as talk about thing-candidates *reduces* to talk about things, this setup should be so far relatively unobjectionable. Existential claims about candidates are not existential claims about the furniture of the universe. Rather, such claims require no more than that there are or can be things, of the sort with which we began, that have such-and-such categorical features at such-and-such spacetime points. The location for controversy is at the question whether there *are* things with the properties of the different candidates.

At this stage, it is an open question whether the statue is distinct from the clay, whether there are point-sized or scattered objects, whether things are temporally extended, and so forth. So one might worry that this thing-candidate setup begs questions against certain accounts of things. But the idea is precisely to *set up* such questions. Perhaps there will be problems of vagueness or ambiguity about which thing-candidates have BEING THE EIFFEL TOWER or BEING THE OUTBACK.[4] Suppose these

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[3]Apologies to Kaplin, "Transworld Heir Lines."

[4]It is important to avoid a certain ambiguity. For any categorical property c, one can make sense of its *necessitation*, BEING NECESSARILY c which a thing has iff it has c in every world, and its *essentialization*, BEING ESSENTIALLY c which a thing has iff it has c in every world where it exists. One can also make sense of the *categoricalization* of such modal properties so that c is the categoricalization of BEING NECESSARILY c and of BEING ESSENTIALLY c. But it may not be clear whether BEING THE EIFFEL TOWER,

are somehow resolved. Still, I do not suggest that every candidate corresponds to something people would ordinarily recognize. Maybe one is ordinary; maybe Martians would recognize it as "ordinary" though we do not. But it does not matter. What matters that we are in a position to specify all the things there are supposed to be for each of the theories to be described. The many thing-candidates are appropriate for this end. Similarly, some thing-candidates are temporally as well as spatially extended. But I do not mean to decide for a four-dimensional over a three-dimensional account of things. Given the many thing-candidates, there is sure to be a reasonable correlation between thing-candidates and the things a three dimensionalist thinks there are. The main outlines of my discussion will remain on a "three-dimensionalist" translation; so I prefer to allow that three dimensionalism may be right, and to remain neutral on this issue. Also, as in the previous chapter, I do not want to engage in controversy over physics. Maybe, for whatever reasons, there are problems about point instantiation of properties. Still, in some sense, physics has to be telling us about the way ordinary things are at various spacetime locations. So let the account of ultimate reality be the true one. Then, subject to what we may think of as simplifying assumptions, this discussion begins where that one leaves off.

So the idea is that, on any account, the properties of things are like the properties of some thing-candidates.

  CT Any thing has just the properties of some thing-candidate.

Let us identify (or lump together) things that cannot differ. Then there is at most one thing with the properties of any thing-candidate. Given our derivation of the categorical and modal properties of candidates from the categorical and modal properties of things, CT should be acceptable from different initial theories of things. But also whatever the initial theory, CT should be compatible with alternate theories of things. So, for example, there are resources in van Inwagen's theory to understand the content of the claim that biceps are things.

Essence-Candidates. We have presented essences as properties bearing a special relation to things. In the simplest case, $\Box \forall x (Ex \leftrightarrow x = a)$. Necessarily a has essence E; and necessarily anything with E is a. This requires that essences be uniquely associated with things. As a condition on essences,

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say, includes some modal elements and so whether it is a categorical property, the essentialization of a categorical property or something else. In general, I suppose that modal elements are explicit or, alternatively, that properties as BEING THE EIFFEL TOWER are already categoricalizations.

EC If a categorical P is such at every world just one bound-candidate has it, then P is an *essence-candidate*.

If P is an essence-candidate then, at any world, if some thing thing-candidates have P, they overlap on a single bound-candidate — and so have all their categorical properties in common. And for any essence-candidate E, there is a unique thing-candidate that has E in every world.[5] For simplicity, identify (or lump together) essence-candidates such that in no world does a thing-candidate have one but not the other — thus we will not distinguish BEING QUINE from, say, BEING A ROUND SQUARE OR QUINE. Then not only is there a unique thing-candidate for each E, but for any thing-candidate t, there is at most one essence-candidate E such that t has E in every world.[6]

The idea is that any essence is an essence-candidate. And essences are matched to things. So,

ES If E is an essence, then there is a thing with the properties of a candidate that has E in every world.

With ES, any essence E is such that a unique thing has it in every world. To see this suppose E is an essence; by ES there is a thing t with the properties of a candidate that has E in every world; by EC there is at most one such candidate; and with CT there is at most one thing with the properties of the candidate. In addition, since there is at most one essence-candidate that the thing-candidate has in every world, E is the only essence t has in every world.

According to the essentialist, some properties are essences. The non-essentialist denies this. As stated, there is room for the essentialist to hold that some things have essences and others do not. This will not be important for our discussion and we shall not worry about it. Still, it is likely that the essentialist holds also that every thing has just the properties of a candidate with some essence E in every world. Then for any thing t there is a unique E that it has in every world.[7] In this case, there is a one-one map from essences to things. At any rate, the essentialist solution applies to things with essences.

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[5]Suppose thing-candidates c_1 and c_2 have E in every world but $c_1 \neq c_2$. Then there is a world w where the categorical of properties of c_1 are distinct from the categorical properties of c_2. But c_1 and c_2 both have E at w; so by EC, the categorical properties of c_1 are the same as the categorical properties of c_2 at w. This is impossible: $c_1 = c_2$.

[6]Suppose c has E_1 and E_2 in every world but $E_1 \neq E_2$. Then there is some w and a such that at w, a has E_1 but not E_2, or E_2 but not E_1; consider the former — since both c and a have E_1 at w, by EC, c and a have all the same categorical properties; but c has E_2 at w; so a has E_2 at w; and similarly in the other case. This is impossible: $E_1 = E_2$.

[7]Suppose t is a thing; then it has all the properties of a candidate with some E in every world; with EC, E is the only essence the candidate has in every world; so E is the only essence t has in every world.

So the essentialist recovers an account of the things to which her solution applies by identifying the essences. Suppose some thing-candidates actually have just the categorical properties of the statue and the clay. If BEING THAT CLAY is an essence, then the clay is a thing with all the properties of a candidate that has BEING THAT CLAY in every world; if BEING THAT STATUE is an essence, then the statue is a thing with the properties of a candidate that has BEING THAT STATUE in every world. So we identify things by identifying the essences. Insofar as these two essences come apart in some worlds, there are worlds where the statue and the clay are distinct, so that the the modal properties of the statue and the clay are not the same. Of course, if one but not the other, or neither (!) of these is an essence, then the essentialist may deny that the statue and/or the clay are things. So far, then, it is an open question whether there are essences, and if there are, which properties they may be.

Of course, I do not claim that everyone, or even that anyone, will grant that every essence-candidate is an essence proper. BEING THE INVENTOR OF BIFOCALS as well as BEING BENJAMIN FRANKLIN are essence candidates. Similarly, BEING THE PRESIDENTS OF THE UNITED STATES is an essence-candidate — one actually had by a spatially discontinuous thing-candidate spread over more than 200 years. The INVENTOR and PRESIDENT properties are not "ordinary" essences, so not all essence-candidates are ordinary. Again, what I do suggest is that one way of thinking abut our range of thing theories is to think of the theories as differing about which, if any, of the essence-candidates are essences.

2.2.2   A Taxonomy of Theories

Either some essence-candidates are privileged relative to others, or they are not. If none are privileged relative to the rest, then none are essences, or all are. And if there is privilege, then there is room for both objective and non-objective modes of privilege. If all candidates are essences, we end up with very many (!) things in the same location. If some essence-candidates are privileged, multiple things in the same location or the "explosion" of things beyond ones we ordinarily think there are might be moderated or entirely avoided — but the privilege itself calls for account. And if there are no essences at all, we are left without the essentialist solution to the *de re* problem. This sets up three main divisions for our approaches to thinghood. The non-essentialist says no essence-candidates are essences, the essentialist that some are. The mad-dog essentialist allows that all essence-candidates are essences, the moderate essentialist says that not all are. And the moderate genuine essentialist thinks selection among essence-candidates is objective, the moderate pseudo essentialist that it is not.

Theories of Things

essentialist   non-essentialist

mad-dog   moderate
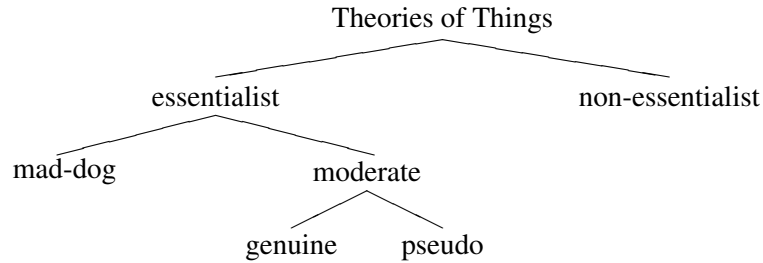
genuine   pseudo

Thus there are mad-dog essentialim, moderate genuine essentialism, moderate pseudo essentialism, and non-essentialism.

Mad-Dog Essentialism.   This view combines *essentialism*, on which some properties are essences, with the *mad-dog* claim that every essence-candidate is an essence. Suppose BEING THIS STATUE and BEING THIS CLAY are essences and some world-bound candidate actually has them. Then corresponding to a thing-candidate with BEING THIS STATUE in every world, the statue is a thing with all the properties of the candidate, and corresponding to a thing-candidate with BEING THIS CLAY in every world the clay is a thing with all the properties of the candidate. Insofar as the thing-candidates actually overlap on a single bound-candidate, the statue and clay are categorically the same. But the statue and the clay differ modally insofar as the one has BEING THIS STATUE in every world and the other does not. Thus a simple solution to the statue-clay case is available: there is no problem with the indiscernibility of identicals, precisely because s and c are not identical.

In this case, the essentialist solution requires revision. If the statue and the clay have their categorical properties in common, they have BEING THIS STATUE and BEING THIS CLAY in common — where each has one necessarily, the other not. Since the thing c has BEING THIS STATUE it is not the case that necessarily a thing has BEING THIS STATUE iff it is s, that $\Box \forall x(x$ is this statue $\leftrightarrow x = s)$. And similarly it is not the case that $\Box \forall x(x$ is this clay $\leftrightarrow x = c)$, or in general that $\Box \forall x(Ex \leftrightarrow x = a)$. However it remains that necessarily, a thing has E iff it has all its categorical properties in common with a — where $x \approx y$ just in case x and y share categorical properties, that $\Box \forall x(Ex \leftrightarrow x \approx a)$. Where E is the essence had by a in every world, anything with all the same categorical properties as a has E. And with EC, anything with E has all its categorical properties in common with a. Then, for categorical P, from $\Box \forall x(Ex \rightarrow Px)$, again it follows that $\Box Pa$.

The mad-dog essentialist avoids difficulties associated with selection among essences by the simple expedient of declining to select among them. Then very many things may share categorical properties! There are the statue and the clay, but

perhaps also things corresponding to BEING THIS STUFF, BEING THESE ATOMS, BEING IN THIS SPATIOTEMPORAL LOCATION, or whatever — all categorically the same but modally different. Without restriction on essences, it seems impossible to separate a thing from its essence. Consider this analogy: By extensionality, a set x is identical to a set y iff x has the same members as y. So, for example, $\{\phi\}$ cannot remain identical to $\{\phi\}$ and be "changed" into $\{\phi, \{\phi\}\}$ because being the latter is being a different set from the former. Switching members switches sets; thus it is natural to think that the members of a set are essential to it. Similarly on the mad-dog view, a thing is distinguished from the full range of other things only insofar as it is linked to a unique essence, and so has the associated modal properties. Switching essences switches things; thus it is natural to think that a thing cannot exist apart from its essence — and so that a thing cannot be decoupled from its essence. Given this, the mad-dog view seems positioned to satisfy MP, according to which things actually have objective modal properties.[8]

We shall return to a related account for the statue and clay. However, for now, there are reasons to worry. Perhaps it will be enough to make a couple of related points about grounding. So far, we have imagined that the statue and the clay are categorically the same but modally different. So AC, according to which modal properties have an actual categorical ground, fails. It is correspondingly difficult to understand how the statue has some modal properties and the clay others. Suppose some thing actually has BEING THIS STATUE and BEING THIS CLAY; these are based in straightforward modifications of the thing. But it is not clear how a thing has one of these properties essentially and the other not — or even how there are different things to have one essentially and the other not. For the view turns essentialism on its head: The idea was to ground modal differences in essences had by things; according to essentialism, the "shape" of a thing as it crosses worlds is supposed to depend on essence. But on this mad-dog view, there exist different things to have different essences only insofar as the things are shaped differently across worlds. Apart from these different "shapes" across worlds, there are no different objects to have the different essences.

Suppose, then, that we retain AC, and somehow the statue actually differs from the clay. Then there must be some categorical difference between the statue and clay to make room for modal difference. Perhaps the view is that thing-candidates

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[8]Yablo, "Identity, Essence, and Indiscernibility," advocates a mad-dog view, or rather one that allows multiple things in the same location. Compare Wiggins, "On Being in the Same Place at the Same Time," and Wiggins, *Sameness and Substance*. Michael Burke, "Copper Statues and Pieces of Copper," gives many references and goes so far as to call the "multiple thing" strategy "standard" for statue-clay cases.

corresponding to the statue and the lump, though qualitatively the same, actually differ in some *non-qualitative* way. On this ground the one stands in a special relation to BEING THIS STATUE and the other to BEING THIS CLAY. About this, first, we may doubt whether such non-qualitative differences are categorical. Grant that objects in a universe of two-way eternal recurrence or with just qualitatively identical brass spheres differ non-qualitatively. Still, with Adams, we may hold that the foundation for such difference is in the actual distribution of qualities. So, "God can create a woman of such and such a qualitative character. And when he has done so, she is an individual and has a thisness, which is the property of being her; and there may be non-qualitative possibilities regarding her. But that property and those possibilities are parasitic on her actual existence" (Adams, "Actualism and Thisness," 10). Insofar as the statue and clay are qualitatively the same, if non-qualitative properties are founded in ones that are qualitative, then there will not be *different* non-qualitative properties corresponding to the statue and to the clay.

Still, suppose that non-qualitative differences between the statue and clay do not depend on the actual distribution of qualities: perhaps they are primitive — then there might be room for different non-qualitative properties of the statue and the clay. Adams wonders how such *haecceities* sustain certain features as opposed to others, as BEING A PERSON rather than BEING A MUSICAL WORK. He raises this as a question about a single thing across worlds. This is a good question. The difficulty is particularly acute in this case: The non-qualitatively different haecceities do not seem adequate to explain how the statue ends up with one set of modal properties, and the qualitatively identical lump with another. So we have a problem, which we may see as related to AC, about an inadequate categorical ground. So it is doubtful whether there is a categorical ground for multiple things and, granting that there is, whether it is adequate to sustain required modal properties. (Compare Zimmerman, "Theories of Masses and Problems of Constitution"; Burke, "Copper Statues and Pieces of Copper"; and Bennett,"Spatio-Temporal Coincidence and the Grounding Problem.")

Moderate Genuine Essentialism.   Within the essentialist framework, though, it is possible to alleviate some of this pressure on grounding by being more selective about the essences. In this way, one might hope to avoid the consequence that things may agree categorically and differ modally. A moderate essentialist view combines essentialism, on which there are essences, with the *moderate* claim that not every essence-candidate is an essence. The view avoids objections from before if no bound-candidate has multiple essences at its world. Then different things differ categorically: If a single bound-candidate has at most a single essence at any world, say, BEING THIS STATUE or BEING THIS CLAY, then there is at most one thing, the statue or the clay, with

the categorical properties of the candidate. And the full essentialist solution is up and running. In this case, essence tracks identity. So where a is associated with E, $\Box\forall x(Ex \leftrightarrow x = a)$, and $\Box Pa$ iff $\Box\forall x(Ex \rightarrow Px)$.

For a view of this kind, one might focus on properties of the sort BEING IN THIS SPACE-TIME LOCATION, or BEING THIS STUFF as the only properties that are essences.[9] On a more traditional approach, one might try for a mixed collection of properties, say, BEING THIS PERSON, BEING THIS CHAIR, and so forth. But on any such account, not both BEING THIS STATUE and BEING THIS CLAY are essences.[10] So, seemingly, not both FS according to which s cannot be as flat as a pancake and FC according to which c can be as flat as a pancake are true. Suppose BEING THIS CLAY is the essence. Then the thing exists in a world just in case BEING THIS CLAY is instantiated there. Plausibly, there is a world where BEING THIS CLAY is instantiated in something as flat as a pancake. So FC is true. As a claim about the *thing*, however, FS is not — for we have allowed that the thing can be as flat as a pancake. Similarly, if BEING THIS STATUE is the essence, FS is true and FC seems not. Or perhaps BEING THIS STUFF is the essence, and we shall have to allow that the thing could have been a scattered object. Or whatever.

Suppose that the essence is BEING THIS CLAY, and that it is therefore possible for the thing to be as flat as a pancake. If FS is true, any view that denies it is mistaken. So it is natural for the moderate essentialist to offer some account of FS according to which it is not, strictly speaking, about the thing. One option is to suggest that reference for relevant terms switches so that FS is a true claim about some fictional object, process, or the like.[11] Then, as for the mad-dog essentialist, there is no problem with the indiscernibility of identicals because $s \neq c$. In *Ontology, Modality, and the Fallacy of Reference*, Michael Jubien suggests a strategy that we shall find useful below. In effect, he appeals to essence-candidates that do not qualify as essences, and analyzes the claim that necessarily s is not as flat as a pancake as a *de dicto* statement of the sort, $\Box\forall x(x$ is-the-statue $\rightarrow \neg(x$ is as flat as a pancake$))$.[12] On this basis, as with the inventor of bifocals, there is no problem about the indiscernibility of identicals because no object is being tracked from world to world. At any rate, corresponding

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[9]Heller, *The Ontology of Physical Objects* suggests a position of the former sort, and Jubien, *Ontology, Modality, and the Fallacy of Reference* one like the latter. Van Inwagen, *Material Beings* develops another moderate account. None express their position this way.

[10]This is so, not only when the the properties are, but also when they can be, had by a single thing-candidate. So, for example, if the clay actually survives the statue — where the story *might* have been as I tell it, then not both BEING THIS STATUE and BEING THIS CLAY are essences.

[11]Some solutions of this sort are sketched and their viability defended in Zimmerman, "Theories of Masses and Problems of Constitution." Heller's appeal to "conventional" objects in *The Ontology of Physical Objects* seems a relatively developed proposal along these lines.

[12] This will do for our purposes; however in his more recent *Possibility*, Jubien offers a more sophisticated, though equally *de dicto*, analysis of such claims.

to the moderate essentialist's selecting among essence-candidates, is some selecting among the formally *de re* claims we might have been inclined to make: either some are false, or some are analyzed into something other than straightforward *de re* modal claims about things.

Insofar as selecting essences from among essence-candidates avoids the consequence that things may differ modally without categorical difference, moderate essentialism avoids corresponding objections brought against mad-dog essentialism. But selecting essences from among essence-candidates raises problems of its own. Along with moderate essentialism, the moderate *genuine* essentialist accepts that selection of essences from among essence-candidates is objective. But it is natural to worry that any selecting among essence-candidates is arbitrary. If the view is to accommodate a response to the problem of modality, there must be a ground for the selection among essences, and so for the required *de re* principles. But what selects properties of the sort BEING THIS STUFF over BEING IN THIS SPACE-TIME LOCATION? Or what selects BEING THIS STATUE over BEING THIS CLAY? A thing, presumably, actually has many essence-candidates. Insofar as each is sufficient to drive an account of things and *de re* modality, each seems an equally strong candidate for essencehood. Say there are two murders, differing only that one is a stabbing and the other a strangulation. Insofar as we think the difference between stabbing and strangulation is morally irrelevant, we are under pressure to hold that the moral evaluation of the events is the same. No categorical property makes one better than the other. And similarly for modality. Insofar as we think there is nothing *modally* relevant distinguishing among the various essence-candidates, there is reason to think that moderate genuine essentialism is false — and perhaps pressure in the direction of mad-dog essentialism. No categorical property makes one candidate necessary and the other not.

Of course, one might hold out for the primitive moral fact that, say, all other things being equal, strangulation is better than stabbing; and similarly, one might hold out for a primitive modal fact that some properties are essences and others are not. However, in the face of AC, these proposed primitives appear mislocated so as to render it mysterious that things should have some modal properties rather than others. It is tempting to respond, "but a primitive is brute and so *without* explanation." But, again, the point from AC is that a theory must properly locate its primitives relative to others. Given a distribution of categorical properties, the concern is that what there actually is, that is everything, is not of the right sort to support these supposed modal facts.

Moderate Pseudo Essentialism.   However, Jubien, at least, develops a view which is not genuine essentialist. In places he suggests that the selecting of essences from

among essence-candidates is conventional (*Ontology, Modality, and the Fallacy of Reference*, §1.1, §2.6; but compare *Possibility*, §1.5). Thus this position is moderate pseudo essentialist. In addition to moderate essentialism, the moderate *pseudo* essentialist accepts that selection of essences from among essence-candidates is not objective. Perhaps the pseudo essentialist reasons as follows: Either the world selects certain properties as essential to things or it does not. If it does, then some genuine essentialist position is right. If it does not then things are "multiform." A thing may have the properties, BEING THIS STATUE, BEING THIS CLAY, BEING THIS STUFF, or whatever, but the point of calling a thing "multiform" is that none of these is singled out by the world as essential to it — or as uniquely determining what it is to be it. *Prima facie*, this is a problem: It is by association with a unique essence, that the essentialist has so far been able to account for a thing's modal properties. And if a thing tracks across worlds at all, the tracking itself selects a unique essence — the one had by *it* in each world where it exists. So, on this view, things do not by themselves track across worlds. Thus there is a question about how things have their modal properties. But the moderate pseudo essentialist accepts that things are, in fact, linked with essences. Perhaps it is convention which supplies that missing element that justifies the selection of just one essence from among a thing's essence-candidates. Given this, proceed as before: On Jubien's view, essences are properties of the sort, BEING THIS PARTICULAR STUFF. The stuff, that is, the thing, which actually is (predicatively) this statue and this clay, might have been scattered; so it is appropriate to say of it that possibly it is a scattered object; but one would not want to say that something scattered could have BEING THIS STATUE or even perhaps, BEING THIS CLAY. And Jubien gives sentences like FS and FC *de dicto* readings of the sort, "necessarily, whatever is-the-statue (or is-the-clay) has such-and-such features." Of course, other moderate options are available.[13]

In this case, what people say may itself count as an actual categorical ground. If it does, AC is satisfied. But, again, there are reasons for us to worry. First, anyone who thinks that a tablet with BEING NaCl, is soluble because of its BEING NaCl, or that a person with ARISING FROM SUCH-AND-SUCH GENETIC MATERIAL arises from it essentially because of her ARISING FROM SUCH-AND-SUCH GENETIC MATERIAL, is likely to think that the moderate pseudo essentialist mislocates the actual categorical ground: the ground

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[13]Burke, "Preserving the Principle of One Object to a Place" advocates a view which seems moderate in spirit but does not fit neatly into my categories. To accommodate his position, I would have to speak, not of properties which are essences *simpliciter*, but rather of properties which are essences *at* worlds. Then a thing exists at every world where the property to which it is linked is an essence. I prefer to avoid this complication. Burke does not declare for a pseudo over a genuine position. But he is clear that his view is at least compatible with a pseudo stance.

is to be found in the BEING NaCl and ARISING FROM SUCH-AND-SUCH GENETIC MATERIAL, rather than in what people say or do. Whatever one says about this, on the moderate pseudo essentialist view, it is not the case that things have objective modal properties; so MP fails. Given a particular thing, on the moderate pseudo essentialist view, it is not objective that one property is its essence rather than another; so it is not objective that the thing has certain modal properties rather than others; so MP fails. MP is, in effect, another constraint on grounding. On the moderate pseudo view, there may be a ground, but not a ground of the sort to satisfy MP.

Non-Essentialism. Problems about grounding the selection of essences from among essence-candidates disappear if there are no essences. So, finally, let us consider a non-essentialist view. Then there are no problems about selecting essences from among essence-candidates, and things are multiform. In this case, it is not merely that things do not by themselves track across worlds, but rather that things do not track across worlds at all. If a thing is linked to no essence, there can be no fact about which things are it in other worlds. If there is a fact about which things are it in different worlds, then there is an essence to which it is linked — the one had by *it* in just those worlds (I assume properties are abundant enough that there would be some such essence-candidate.) If things do not track across worlds and supposing, as we have, that different things can differ categorically, different things in fact differ categorically. So the the non-essentialist avoids the mystery of mad-dog essentialism insofar as no more than one thing corresponds to any bound-candidate. But all this leaves the non-essentialist with serious questions about whether things have modal properties at all.

Perhaps the non-essentialist is inspired by the suggestion that if "alternative" accounts are adequate to accommodate some formally *de re* sentences, then maybe some such account is adequate to accommodate them all. So, for example, perhaps reference shifts so that *de re* modal claims are in general about fictional objects; or perhaps formally *de re* modal claims can in general be analyzed into those that are *de dicto*. Either way, the non-essentialist seems to require *some* associations with essence-candidates for modal truth — the fictional object "has" one, the *de dicto* analysis incorporates one, or whatever. Perhaps a relation between essence-candidates and reference is supposed to do the job. Grant that some such account makes sense of FS and FC. Still, there is a problem about modal properties. So far, non-essentialism seems to be a more-or-less sophisticated denial of the claim that things have modal properties, rather than a view according to which MP, things have objective modal

properties.[14]

Thus we encounter a bulge in the carpet associated with grounds for modal properties. Essentialism makes room for modal properties, but has problems with a ground; non-essentialism shifts difficulties into difficulties for modal properties themselves. Shifting the account of things shifts the bulge, but does not seem to push it down. So far, then, the problem about modality resists solution. In the following, I develop a pair of responses on which essences have an independent or explicit role for *de re* modality. These accounts are structurally parallel, and one leads naturally to the next, though they are at opposite ends of our spectrum: the first for *proper* things is non-essentialist, and the second for *ordinary* things is mad-dog essentialist.

2.3 Proper Things

Assume a non-essentialist position on which there is a *proper* thing corresponding to each bound-candidate, and there are no essences. With hollow blackboard symbols to range over proper things, say the non-essentialist thinks there are connections between reference and essence-candidates so that FS is equivalent to something like $\Box \forall x\, (x$ is-the-statue $\to \neg (x$ is as flat as a pancake)). Given this, a *de dicto* solution may seem sufficient for the whole of the problem of modality. In this part, I accept this non-essentialist position. However I go on to argue that the associated solution is not *de dicto*. Rather its underlying form is *de re*. On the non-essentialist position, there are no essences. However, essence-candidates may be "essence-like" insofar as they set up cross-world "same-statue," or "same-clay" relations. The result is a modified version of essentialism.

I begin by addressing grounds for modality. To connect with the statue-clay case, I will then have to turn to some considerations about reference. And, because of controversy likely to be excited by the resultant position, I conclude the section with some remarks about identity.

2.3.1 Grounds for Modality

Let us take it as given that it is possible for a thing to have BEING THIS CLAY without the precise location or shape this clay actually has; similarly, it is possible for a thing to be this clay without being this statue, and, insofar as the clay might outlast the statue,

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[14]Familiar occupants of the non-essentialist position are those that, as above, deny that things have *de re* modal properties and adopt some alternative account — somehow linking the applicability of modal expressions with the manner in which things are designated. See, for example, Quine, "Reference and Modality."

for one thing to be this clay and another to be this statue. All the different approaches to thinghood allow this much, and differ only on how to allow for it. On a mad-dog essentialist view, perhaps both BEING THIS STATUE and BEING THIS CLAY are essences; on a moderate view, perhaps one is an essence and another figures in an alternate account; on a non-essentialist account, both may figure in alternate accounts. But on any of the accounts, there are some facts about whether things across worlds count as the same statue or the same clay, perhaps not the same as their being the same *thing*. Also, just as there may be a BEING TALLER THAN JIM ACTUALLY IS which applies across worlds, but depends on Jim's actual height, so I suggest that BEING THIS STATUE and BEING THIS CLAY apply across worlds but depend on the actual thing. Just as there is a cross-world relation relation associated with BEING TALLER THAN JIM ACTUALLY IS which explains how it depends on Jim, so I suggest that there are cross-world "same-statue" and "same-clay" relations associated with BEING THIS STATUE and BEING THIS CLAY which explain how they depend on the actual thing. As the essentialist thinks there are facts about the cross-world identity of things, so the modified essentialist thinks there are these cross-world relations, and that *de re* modality is grounded in them. Let us turn to this second point.

For any kind K, one expects BEING A K to involve having certain general features and/or standing in relations to individuals of certain sorts, where BEING A PARTICULAR K requires having those general features and standing in those relations to particular individuals. Suppose BEING A PERSON requires having a certain sort of continuity; then for any person, BEING THAT PERSON requires having that continuity; necessarily, anyone who is that person has that continuity. Suppose BEING A PERSON requires having some fixed (genetic) sex; then for any particular person, BEING THAT PERSON requires having a particular sex; necessarily, anything that is that person has that sex. And suppose BEING A PERSON involves having some one mother; then for any particular person, BEING THAT PERSON requires having a particular mother; necessarily, anything that is that person has that mother. From kinds alone then we obtain general principles of the sort,

P1 $\forall x (Px \rightarrow \Box \forall w (SPwx \rightarrow Cw))$

P2 $\forall x [(Px \wedge Fx) \rightarrow \Box \forall w (SPwx \rightarrow Fw)]$

P3 $\forall u \forall v [(Pu \wedge Pv \wedge Mvu) \rightarrow \Box \forall w (SPwu \rightarrow \exists x (SPxv \wedge Mxw))]$

If anything is a person, then necessarily anything that is the same person as it has the right sort of continuity; if anything is a person and female, then necessarily anything that is the same person as it is female; and if *u* and *v* are persons and *v* is the mother

of u, then necessarily if w is the same person as u there is some x such that x is the same person as v and x is the mother of w.[15] Formally, P1, P2, and P3 are *de re*. But notice: having assigned some actual thing to a variable or constant, we do not relocate that thing in other worlds; rather, we locate other-worldly things which stand in the "same person" relation to the actual one. Thus, again, given our characterizations of *de re* modality, there is a question about the sense in which we have identified a legitimate modal feature of a thing.

The suggestion that we have identified such a feature is reinforced by grounding considerations. Grounds for P1 - P3 may be like those associated with the *de dicto* solution. It is part of the structure of the kind property that such relations hold. But the principles make what it is to be a particular κ follow from the nature of that kind property *and* nonmodal categorical features of the thing. So, for example, it is because some thing is a person, that necessarily whatever is that person has the right sort of continuity: If Pa, from P1, $\Box\forall w(SP wa \to Cw)$. And, unproblematically, if $a = b$, then $\Box\forall w(SP wb \to Cw)$. Similarly, it is because a person actually has some sex, that having that particular sex is part of what it is to be that person. And it is because a person actually has a particular mother, that having that particular mother is part of what it is to be that person. Perhaps it is of the nature of BEING MORALLY DEPRAVED that anyone who plucks out the eyes of innocent children is morally depraved — this much may depend on the property alone; still, anyone who plucks out the eyes of innocent children IS MORALLY DEPRAVED, with their depravity firmly grounded in the categorical way they are. Similarly, from principles like P1 - P3, a thing's BEING SUCH THAT NECESSARILY ANYTHING WHICH IS THE SAME κ AS IT IS P might depend on the categorical way that thing is, and so count as a legitimate *de re* modal feature of the thing.[16]

One might object that u in one world cannot be the same κ as v in another, unless u and v are κ's and $u = v$. So u in one world cannot be the same statue as v in another unless both u and v are statues, and $u = v$. But then we require identity across worlds — and this is something the non-essentialist cannot provide. I think, however, that this reverses the proper order of things. The essentialist gives an account of identity across worlds in terms of some essence property; that is, u in w_1 and v in w_2 have the property, and on this basis, the essentialist rules that $u = v$. We allow the

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[15]Related principles are suggested by Kripke, *Naming and Necessity* (111-115), and developed by others; see especially, Sidelle, *Necessity, Essence, and Individuation*. In chapter 3, I develop the suggestion that such principles enable an account of modal claims about things that could, but do not, exist.

[16]A three-dimensionalist might respond similarly to puzzles about, for example Tibbles the cat: In the past, something was the same cat as this and had a tail. But, in the past, nothing which was the same puss (that part of a cat which does not include its tail) as this had a tail.

same, only less: u in w_1 and v in w_2 have the property — only, given that no essence-candidate is privileged, this is not sufficient for identity. But the cross-world relation determined by the property remains. This is the basis for the suggestion that our view "modifies" essentialism. The relations hold up no less under non-essentialism as under essentialism. So far then, our account of the relations is parasitic on the other account: insofar as essentialist treatments of the properties make sense, ours does so as well.

Given the cross-world relations, the *de re* modal properties result as described above. But, again, one might object that the resultant modal properties cannot derive from BEING THE SAME K AS THIS, insofar as nothing on our account is a K. That is, on our account, a thing may have whatever is required categorically of BEING THIS CLAY or BEING THIS STATUE, and things in different worlds may have these insofar as they stand in the SAME K relation to the actual thing. This is the basis for the modal properties the non-essentialist allows things have. But, the objection goes, nothing is the statue or the clay in any robust sense — for the thing proper is not such that there is a fact of the matter about which things are it across worlds. And one might think that nothing is a statue or clay apart from being essentially a statue, or essentially clay. Put this way, the point is right: On the current view, if ordinary things have some properties essentially, then no proper things are ordinary. We will return to this question about the recovery of ordinary things in the next section. For now, if our modified essentialism is to accommodate claims of the sort FS and FC, then FS and FC must be understood given just things proper.[17]

And we are in fact on the verge of a response to the case of the statue and the clay. It may be that BEING A STATUE requires a certain continuity of shape, so that if a thing is a statue and has a certain shape, then necessarily anything which is that statue has (roughly) that shape. But BEING SOME CLAY may require no such continuity. Thus, where some one thing is both a statue and some clay, it may be that necessarily nothing is the same statue as it and is as flat as a pancake, though possibly something is the same clay as it and has that shape. These are perfectly consistent, and may seem straightforward *de re* modal properties of the thing. Thus we take a significant step toward a direct response to the problem about grounds for modality with which we began. Observe, however, that it is not yet an account of FS and FC. For on the modified essentialist account, modal claims are *kind-relative*. Insofar as the view is

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[17]See note 4. One might object that there is at least a problem about expression insofar as we say a thing has 'BEING THIS K' though no thing proper is an ordinary K. But observe: while BEING THIS K in fact results in an ordinary K on essentialist views, the very same property does not so result under non-essentialism. We seem stuck with the property name from the dominant essentialist views. Further, we shall have use for the locution as descriptive again in the next section. So it seems best to retain the locution, given that we have correctly sorted out just what work the property does on our non-essentialist account.

non-essentialist at bottom, there is no fact of the matter about whether a thing by itself is possibly this or necessarily that. So in order to make direct contact with FS and FC, I need to say some things about names — and in particular, about whether these expressions including s and c deliver the sort of content required to make claims of the sort I have argued are true.

2.3.2   Naming and Reference

Just as each of the accounts of things requires some work from essence-candidates, so each of the views requires some associations between essence-candidates and reference. And some associations between reference and essence-candidates may seem to be part of the data. If FS and FC are true, s works differently from c; somehow, s makes BEING THIS STATUE matter for FS, where BEING THIS CLAY matters for FC. And similarly when modal claims seem to fail. It is natural to ask whether *this statue* could have been as flat as a pancake, and to ask whether *this clay* or *this stuff* could have had that shape. In contrast, there is something odd about the question of whether the thing that is this stuff, that is, the thing that is this clay, that is the thing that is this statue, could have been as flat as a pancake. Presumably, there is no problem about whether it is red or two feet tall. In the modal case, however, the thing is identified — and we are assuming a non-essentialist account on which there is just the one thing — but without recourse to any one essence-candidate, and this seems to explain the oddity. Naturally, the different approaches to things have different ways to account for the data. However, insofar as each explains the full range of data (and pending full accounts of reference), it may be that each introduces associations between essence-candidates and reference: the mad-dog essentialist to distinguish things that agree categorically, the moderate essentialist and non-essentialist to distinguish among different, maybe alternative, accounts. Insofar as each requires such associations, none is at a relative disadvantage for doing so. Still, it is hardly clear how the associations are supposed to work. Indeed, the proposal that there are such associations seems to turn around the received Kripkean picture, on which things independently bear modal properties, and names need merely be tags. Thus, post-Kripke, how can there be associations between reference and essence-candidates of the sort modified essentialism (or any of the accounts) seem to require?

At one level, our account is perfectly consistent with the suggestion that things independently bear modal properties, and names are mere tags. Thus, where some a is (categorically) a statue and some clay, it may be that necessarily nothing is the same statue as a and is as flat is a pancake, and possibly something is the same clay as a and is flat as a pancake. And for $a = b$, we have substitution so that necessarily

nothing is the same statue as b and is as flat is a pancake, and possibly something is the same clay as b and is flat as a pancake. So far, so good. The problem is that, if expressions of the sort FS and FC are true, ordinary names like s and c must be more than mere tags. And Kripke's claims are typically taken to apply precisely to names like s and c.

In "Actualism and Thisness," Robert Adams suggests thinking of a thing's *thisness* — the property of BEING IDENTICAL WITH IT — as a pair that has, as one member, the thing and as the other the relation of identity. Whatever one thinks about this identification, it is natural to treat BEING THIS STATUE, BEING THIS CLAY, and the like, as corresponding to pairs $\langle a, \kappa \rangle$ this time of thing and kind property. The first member of the pair *is* the thing, and the second is the kind — BEING A STATUE, BEING SOME CLAY, or whatever — so that the pair does not depend on language. Plausibly, though, necessary for reference to the pair, and for reference to the corresponding property, is reference to the thing. BEING THIS STATUE is one thing and BEING THAT STATUE is another; *this* statue has BEING THIS STATUE, and *that* statue has BEING THAT STATUE. Similarly, if this is a world of two-way eternal recurrence, it is one thing to be this statue, and another to be the corresponding statue in a previous age. The point is not that the property *is* the pair, or that properties somehow have things as "constituents," but rather that reference to the things (or something like it) is required to disambiguate the properties. If this is right, we are in a position to explain the use of names in formally *de re* claims. Even if truth conditions for propositions expressed by modal sentences depend on relations between properties, still, for certain properties — properties "gotten" only by way of things, reference to the things seems required for expressing the relations. But then it may be natural to express such relations by means of a device that includes at least some directly referential component. Of course, even granted that ordinary proper names fill this role, we do not have that names are sufficient for expressing the relations, because getting at these properties requires getting at kinds as well. But perhaps it is not implausible to think that ordinary names do get kinds as well.

Building on Kripke's picture, it is natural to think that kind properties are necessary features of ordinary name baptism and transmission.[18] Say a Δt-*slice* of a thing (relative to some reference frame) is that thing over the span Δt (in that frame). Suppose my mother is naming me. She lovingly says, "Let the person of which that (she points) is a Δt slice be T.R." It turns out that she names a person. But, however

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[18]Treatments differ, however appeal to kinds for reference and essence is familiar. Though what I say differs from each, one might see this part as reworking or recasting motivations that underlie, for example, Gibbard, "Contingent Identity," Geach, *Reference and Generality*, and — with his treatment of "rigid singular categorials" — Jubien, *Ontology, Modality, and the Fallacy of Reference*. Compare Kripke, *Naming and Necessity*, 115-116n58.

unlikely, she might have said, "Let the *stuff* of which that baby slice is now composed be T.R." In this case, with sufficient gesticulation and philosophical explanation, she might have named some stuff — stuff which an interested scientist could determine to be a rock (or fertilizer) at a later time. Apparently, the PERSON and STUFF properties figure in getting the name onto the thing that "spreads out" from the slice in the right way (given the many things of which the slice is a part). Naturally, the issue is not so simple: Kind properties may be suppressed in baptisms. Or one might point and say, "Let that *star* be Hesperus" — and end up naming a planet. Perhaps there is some complex interaction between the world, convention, what people say, and so forth. But, where a kind property just *is* a property that figures in getting a name onto the thing that "spreads out" from a slice in the right way, all I am after is that some kind property is a feature of that process whereby a name is attached to a thing. Perhaps this much is plausible.

Even so, our non-essentialist requires that kinds continue to matter for ordinary usage. First, it is natural to think that name transmission chains trace to baptisms, and to things through or by means of the baptisms. But if baptisms figure in the continued working of names, and kinds in the working of baptisms, then it seems that in some sense kinds continue to figure in the working of names. Modal evidence to the side, it may be possible to motivate this directly. Suppose someone overhears in a market, "Quine is very good" and takes it as an evaluation of some Australian wine; upon remarking to his wife at a later time that he would "try to pick up some Quine," he will surely not have succeeded in referring to Quine — despite his intention to use the word the way it was used by those from whom he heard it. Or consider Socrates just before his death and suppose someone says that Socrates, *the stuff*, is now a scattered object. Perhaps this is so. But perhaps not; perhaps 'Socrates' resists association with stuffhood. At any rate, the ability to block reference (as with the 'Quine' case), the ability to switch reference (as with 'Socrates' on the former interpretation), and even the phenomenon of resistance (as with 'Socrates' on the latter interpretation), are compatible with a model on which kinds continue to matter for reference. Again, no doubt there is a complex interaction between context, convention, history, and so forth. For our purposes, though, it is sufficient that, in modal contexts, use of an ordinary proper name somehow "finds" a kind as surely as it finds a thing. For finding a thing and a kind (which the thing has) is sufficient for finding an essence-candidate, and may thus be sufficient for an account along the lines of modified essentialism.

I am not sure whether this approach to names fits the Frege-Russell picture because it allows that a name has something like a sense, or Kripke's because it makes chains reaching back to things matter. In either case, I do *not* offer an account of how reference works or an analysis of sentences containing ordinary proper names. It is

sufficient that, in effect, ordinary proper names "supply" or "return" objects just in association with corresponding essence-candidates. Where pairs $\langle a, \kappa \rangle$ correspond to essence-candidates as above, let us indicate the association between a name and a candidate by a subscript. So $a_{\langle a, \kappa \rangle}$ returns a thing just in case it has the essence corresponding to $\langle a, \kappa \rangle$, which is to say, just in case, $\Box \forall x (SKxa \leftrightarrow x = a_{\langle a, \kappa \rangle})$; thus $a_{\langle a, \kappa \rangle}$ is a "variable" name attaching to whatever (proper) thing is the same κ as a. Given this, for the postulated connection between reference and essence-candidates, no special *analysis* of $\Box P a_{\langle a, \kappa \rangle}$ is required; rather it *follows* that $\Box P a_{\langle a, \kappa \rangle}$ iff $\Box \forall x (SKxa \rightarrow Px)$. The discussion of kinds and reference only makes room for the suggestion that names do return objects in this way. Also, on the surface at least, our non-essentialist is not subject to the sort of objections Kripke brings against Frege and Russell. One worry concerns the connection between what speakers grasp and what is required for reference. However, insofar as essence-candidates appear only at the "ends" of reference chains, there is no reason to think that speakers grasp them. And kinds might be located at the "ends" as well. A related worry is that descriptions associated with, for example 'Nixon', could characterize persons other than the one that is, intuitively, *that person*. But our non-essentialist is committed precisely to the claim that 'Nixon' attaches to whatever has the property of being that person; so the objection does not get off the ground.[19]

But if this much is right, it is a simple matter to accommodate FS and FC. For if '*s*' is a name which attaches to things so that necessarily, any x is the same statue as this iff $x = s$, then FS follows from our previous result that necessarily, nothing is the same statue as this and is as flat as a pancake. And if '*c*' is a name which attaches to things so that necessarily, any x is the same clay as this iff $x = c$, then FC follows from our previous result that possibly something is the same clay as this and is as flat as a pancake. So FS and FC are true.[20]

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[19]Kripke's own emphasis on kinds in modal contexts is congenial. So, for example he says, "Such terms as 'the winner' and 'the loser' don't designate the same objects in all possible worlds. On the other hand, the term 'Nixon' just is a *name* of *this man*. When you ask whether it is necessary or contingent that *Nixon* won the election, you are asking the intuitive question whether in some counterfactual situation, *this man* would in fact have lost the election" (*Naming and Necessity*, 41, emphasis his; compare 46, 51-52, 57 and especially 112-113).

[20]Insofar as FS and FC attribute *different* properties to the one thing, BEING AN x SUCH THAT NECESSARILY NOTHING IS THE SAME STATUE AS x AND IS AS FLAT AS A PANCAKE, and BEING AN x SUCH THAT POSSIBLY SOMETHING IS THE SAME CLAY AS x AND IS AS FLAT AS A PANCAKE, one might think of this as a fleshed out version of the "predicate switching" strategy suggested in Noonan, "The Closest Continuer Theory of Identity." For a related approach, see Lewis, "Counterparts of Persons and Their Bodies."

2.3.3 Naming and Identity

De re modality though this may be, it is not everything typically associated with quantified modal logic. The modified essentialist offers an account of modal expressions, like $\Box P a_{\langle a, \kappa \rangle}$, whose only individual terms are ordinary proper names. And there is room for modal expressions, like $\Box \forall x (SKxa \rightarrow Px)$ with objectual reference and quantification. But, insofar as there is no fact of the matter about whether a thing a exists in different worlds, there may be no fact about the truth or falsity for expressions, like $\Box P a$, which do not build in which things are under consideration across worlds. (Although not based on a general rejection of modal properties, this seems to be a point of contact with Quine.) One recovers much of ordinary quantified modal logic if restricted to expressions with individual constants of the sort $a_{\langle a, \kappa \rangle}$ and, perhaps, a substitutional quantifier — or, what might come to the same thing, if thing/kind pairs are assigned to variables and individual terms. Still, the necessity of identity does not obtain. It might therefore be objected that the modified essentialist is committed to an unacceptable version of "relative" or "contingent" identity.

It is typically held that $\Box(a = b)$ follows from $a = b$. Reasoning, due to Marcus, is simple: Suppose $a = b$; then a and b have all their properties in common; but a is necessarily a; so b is necessarily a. Depending on the nature of the terms, however, the modified essentialist has different replies. First, for directly referential a, there is a problem about even $\Box(a = a)$. If there is no fact about whether a exists in some world, it is hard to see how there could be a fact about whether a has some property or relation — even one so basic as identity — at that world. Thus there is room to reject Marcus's premise. In this case, however, the modified essentialist does *not* allow that identity is contingent. Rather, the suggestion that identity is contingent or necessary presupposes that things track across worlds. Insofar as this presupposition constitutes a challenge to our assumed non-essentialism, the modified essentialist must resist. Second, from $a_{\langle a, \kappa \rangle} = b_{\langle b, J \rangle}$, it does not follow that $\Box(a_{\langle a, \kappa \rangle} = b_{\langle b, J \rangle})$. On the modified essentialist's view, $a_{\langle a, \kappa \rangle}$ returns an object just in case it has $\langle a, \kappa \rangle$ and similarly for $b_{\langle b, J \rangle}$. Given this, $\Box(a_{\langle a, \kappa \rangle} = b_{\langle b, J \rangle})$ iff necessarily something is the same κ as a iff it is the same J as b, iff $\Box \forall x (SKxa \leftrightarrow SJxb)$ — and $\Box(a_{\langle a, \kappa \rangle} = b_{\langle b, J \rangle})$ is not a *de re* identity statement (relative or otherwise) at all; rather it requires that properties be necessarily coinstantiated. But it is uncontroversial that actually coinstantiated properties may come apart in other worlds; for example, something might be the same clay as this without being the same statue as this. As for 'the first Postmaster General = the inventor of bifocals', then, the indiscernibility of identicals does not apply, and the Marcus argument does not go through.

In fact, the modified essentialist predicts two ways to have $a_{\langle a, \kappa \rangle} = b_{\langle b, J \rangle}$, without

$\Box(a_{\langle a,\kappa\rangle} = b_{\langle b,\jmath\rangle})$. The first is when $\kappa \neq \jmath$. This is the case for the statue and the clay. Though $a = b$, if $\kappa \neq \jmath$, then $\langle a, \kappa\rangle \neq \langle b, \jmath\rangle$, and from $a_{\langle a,\kappa\rangle} = b_{\langle b,\jmath\rangle}$, it does not follow that $\Box(a_{\langle a,\kappa\rangle} = b_{\langle b,\jmath\rangle})$. Suppose we have some reason for moving from $a_{\langle a,\kappa\rangle} = b_{\langle b,\jmath\rangle}$ to $a = b$. Then perhaps it is natural to appeal to some convention proposal, or claim about implied premises, to explain a tendency to think that the kinds are the same, and so to move from $a_{\langle a,\kappa\rangle} = b_{\langle b,\jmath\rangle}$ to $\Box(a_{\langle a,\kappa\rangle} = b_{\langle b,\jmath\rangle})$. So, for example, if there is a hierarchy of kinds such that one kind is selected or assumed for each thing, then from $a_{\langle a,\kappa\rangle} = b_{\langle b,\jmath\rangle}$ it follows that the kinds are the same. On the modified essentialist view, though, no one convention is such that naming must, in some metaphysical sense, proceed by means of it. Maybe *typically* naming is governed by such a convention so that *typically* it is appropriate to move from $a_{\langle a,\kappa\rangle} = b_{\langle b,\jmath\rangle}$ to the conclusion that the kinds are the same. But the statue-clay case suggests that whatever conventions of this sort there may be, they are not inviolable.

The second way to have $a_{\langle a,\kappa\rangle} = b_{\langle b,\jmath\rangle}$ without $\Box(a_{\langle a,\kappa\rangle} = b_{\langle b,\jmath\rangle})$ is illustrated by another puzzle: It is natural to admit that a wooden lectern could have been plus or minus a few of its molecules. Intuitively, however, a lectern with completely different molecules would be a different lectern. Pretend a lectern is the same just in case it retains at least 75% of its molecules. Now consider a series of worlds, where lecterns in adjacent worlds differ by only a few molecules and so are the same, but lecterns in worlds at the endpoints are composed of completely different molecules and so are distinct. Identity is transitive, so something is wrong (compare Chisholm, "Identity Through Possible Worlds"). But the solution is immediate. Consider any two worlds in the series — say *this* one and *that* one. Insofar as BEING THIS LECTERN is grounded in the particular molecules of this lectern, and BEING THAT LECTERN is grounded in the molecules of that one, and the lecterns are composed of different molecules, BEING THIS LECTERN is a different property from BEING THAT LECTERN: BEING THIS LECTERN requires having at least 75% of these molecules, and BEING THAT LECTERN lectern having at least 75% of those molecules. Perhaps the properties are coinstantiated in some worlds; still, there is no reason to think that there are not worlds where they come apart. In particular, a lectern at one endpoint may lack the property of being the one at the other. Perhaps problems of vagueness remain. But, given the modified essentialist's account of 'this κ = that $\jmath$', problems about the necessity of identity and cross-world identification evaporate. Say $a_{\langle a,\kappa\rangle} = b_{\langle b,\jmath\rangle}$; if a and b are in distinct worlds, we do not have that $a = b$; so we do not have that $\langle a, \kappa\rangle = \langle b, \jmath\rangle$, and it does not follow that $\Box(a_{\langle a,\kappa\rangle} = b_{\langle b,\jmath\rangle})$. Of course, in ordinary cases, a and b are actual. If anything has $\langle a, \kappa\rangle$ at a world where a exists, it is a, and similarly for b; so if a and b are actual and $a_{\langle a,\kappa\rangle} = b_{\langle b,\jmath\rangle}$, then $a = b$. In this case, with the addition of a convention or implied premise about kinds, it *is* appropriate to move from $a_{\langle a,\kappa\rangle} = b_{\langle b,\jmath\rangle}$ to both

$a = b$ and $\kappa = \jmath$, and so to $\Box(a_{\langle a, \kappa \rangle} = b_{\langle b, \jmath \rangle})$. Thus the modified essentialist might have no problem with a move from Cicero = Tully to $\Box$(Cicero = Tully), and similarly for other standard examples of the necessity of identity.

The position developed in this part corresponds to some extent with each of the others considered. With the mad-dog essentialist I allow that different essence-candidates may be "created" equal, but with the moderate essentialist that there is at most one thing for each thing-candidate. With the moderate genuine essentialist I allow that things by themselves have modal properties, but with the moderate pseudo essentialist that things are multiform. And, of course, the position is non-essentialist. And there is a response to the mystery with which we began. NG is false, and we may see the modal properties of things as grounded in the categorical way they actually are. It is a point of contact with Quine that there are problems about quantified modal logic, but with his opponents that some objectual *de re* modal claims make sense.[21]

2.4 Ordinary Things

In the previous sections, we encountered a bulge in the carpet associated with grounds for modal properties. Essentialism makes room for modal properties, but has problems with a ground: Mad dog views strand things with different modal properties on the same ground. Moderate views raise questions about privileging some essence-candidates over others. Non-essentialism cannot make sense of "ordinary" things that track across worlds. Our modified essentialism results in modal properties as, BEING SUCH THAT POSSIBLY SOMETHING IS THE SAME CLAY AS THIS AND IS AS FLAT AS A PANCAKE, with kinds built into the modal properties themselves. This recovers much of what we want to say, though it does not yet recover ordinary things that track across worlds, and does not recover standard modal logic with objectual quantification and the necessity of identity. But we are positioned to recover ordinary things within the bounds of grounded platonism: Our account is so far structurally parallel to one that pushes kinds from the account of reference into the account of things themselves. Ordinary physical things result from an association of kind properties together with proper things. On this base, one might recover the full quantified modal logic as applied to ordinary things. And we set the stage for ordinary things that are abstract in addition to ones that are concrete.

The idea is that an adequate ground for ordinary things is just things proper, with some kind properties. The proper thing with BEING SOME CLAY constitutes one ordinary

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[21]It worth considering how the positions considered in this section fit into Michael Rea's excellent taxonomy of options for lump-statue cases from, "The Problem of Material Constitution" See also the "Introduction" from his *Material Constitution*.

thing, and the proper thing with BEING A STATUE another. The kind property supplies a function from the proper thing to the ordinary thing together with its properties — modal and otherwise. Insofar as there is no privileging of essence-candidates, the result is a version of mad dog essentialism. Insofar as the statue and clay have different constituting elements, objections so far raised against the mad dog view do not apply. In this section, I begin by developing this proposal as it applies to ordinary physical things, statues, clay and the like. As we shall see, however, the picture leaves room for things in addition to physical things. Thus I take up objects of other sorts, including events and fictional objects, and suggest that these are no less ordinary than statues and clay. The latter case makes good so much of GP: some ordinary things are abstract. The picture makes room for things which may be the ground for other things, which may themselves be the ground for further things, and so forth. So I conclude with some steps toward a model to represent the way this fits together.

2.4.1 Ordinary physical things

We have so far associated essence-candidates with pairs $\langle a, \kappa \rangle$ of proper thing and kind property. Given a proper thing and a kind property instantiated by it, there is an essence-candidate such that things in different possible worlds are the same κ as the thing. We have taken advantage of such relations for modal claims. But if an ordinary physical thing is one that tracks across worlds, and a proper thing and kind property are sufficient to identify things in different worlds, a proper thing and kind property are sufficient to ground an ordinary thing.

OP An *ordinary physical* thing t is constituted by a kind with a proper thing (a, κ), where κ fixes a function from a to t with all its properties.

This function from a is such that t has essence $\langle a, \kappa \rangle$ (again, recall that the essence is not the pair, but merely corresponds to it). Suppose a proper thing a actually has both BEING A STATUE and BEING SOME CLAY. Then there is one ordinary physical thing constituted by $(a,$ BEING A STATUE$)$ with essence $\langle a,$ BEING A STATUE$\rangle$, and another constituted by $(a,$ BEING SOME CLAY$)$ with essence $\langle a,$ BEING SOME CLAY$\rangle$. The features of the one are a function of the features of a given BEING A STATUE, and of the other a function of the features of a given BEING SOME CLAY. It may be that the statue has all the same categorical features as the clay. However, the statue is not identical to the clay. And, in contrast with the mad-dog view described above, the statue does not have all the same constituents as the clay.[22]

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[22]It is frequently observed that features of artifacts may depend on more than their matter, including intentions of creators and the like (as Baker, *Persons and Bodies*, 39-46). Though it is perhaps poorly

Mechanics. The proposal is that an ordinary physical thing is constituted by a proper thing and kind (a, κ). Of course (a, κ) is no set-theoretic object. An ordinary object is constituted by the proper thing a and kind κ — the notation merely indicates the constituting elements. In this case, constituents need not be parts — especially properties are not usually among the parts of things. From the kind, there is a function from a to the ordinary physical thing. And these functions are such that an ordinary thing with constituents (a, κ) has essence $\langle a, \kappa \rangle$. Because an ordinary thing is constituted by some (a, κ) where κ fixes a function from a to an ordinary thing including its essence $\langle a, \kappa \rangle$, not just any (a, κ) constitutes an ordinary thing. In particular, for an ordinary thing, the output from (a, κ) is (essentially) of kind κ; so it is required that a has κ. Perhaps it is best to say a kind κ sets up a partial function, where objects in its domain are ones with κ. In case a is not in the domain of such a function, (a, κ) does not constitute an ordinary thing. But in any case where a is in the domain of a function fixed by κ, an ordinary thing is constituted by (a, κ). Thus, drawing upon examples from before, if some a has BEING A STATUE, BEING SOME CLAY, BEING SOME STUFF or the like, there are ordinary things corresponding to each, and ours is a version of mad-dog essentialism. So we continue to reject privileging essence-candidates — now, of course, letting them all in, rather than keeping them all out.

These things are "ordinary" at least in the technical sense that they track across worlds. This puts us in a position to recover much, if not everything, typically associated with ordinary things. As suggested in the last section, for naming, it is likely that some association between names and kinds must remain on a mad-dog account, at least to distinguish things that agree categorically. But once names are attached to things, the names may work like mere tags. Thus, insofar as 's' picks out the ordinary statue and 'c' the clay, we have $s \neq c$, where both FS and FC are straightforwardly true. It may be that $s \approx c$, that the statue and clay have their categorical properties in common. But there is a basis for different modal properties, insofar as the statue and clay do not have all the same constituents. In addition, insofar as ordinary things track across worlds, the argument for the necessity of identity is recovered: Given that (names are rigid and) ordinary things track across worlds, $\Box(s = s)$; so with $s = s'$, it is immediate that $\Box(s = s')$. So this much of ordinary modal logic is recovered too.

This approach permits some generalizations. Say $Czx\kappa$ and $CQx\kappa$ when z and Q are an ordinary thing and essence appropriately constituted by x and κ. Then for

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chosen, we are stuck with the example by tradition. For now, let us keep to the simple account: Either set this complication to the side, or change the example. So we might think about the atoms and and the clay, or the the stuff and the clay, and continue as above.

ordinary physical things we expect,

Oe $\forall z \forall Q [\exists x \exists \kappa (Czx\kappa \wedge CQx\kappa) \rightarrow \Box \forall w (IQw \leftrightarrow w \approx z)]$

If z and Q are constituted by x and κ, then necessarily a thing instantiates Q iff it has the same categorical properties as z. So once the consequent is detached, for ordinary physical things the basic mad-dog version of the essentialist solution is in play. As we shall see, this principle applies not just to ordinary physical things, but reappears for ordinary things more generally.

In the case of ordinary physical things, we have already a reasonably robust theory of the relation between ordinary things and the proper things and kinds by which they are constituted. First, an ordinary thing and essence are constituted by a proper thing and kind just in case the proper thing instantiates the kind. Say $K\kappa$ just in case κ is a kind. Then,

O1 $\forall \kappa \forall x (\exists z Czx\kappa \leftrightarrow K\kappa \wedge I\kappa x)$

O2 $\forall \kappa \forall x (\exists Q CQx\kappa \leftrightarrow K\kappa \wedge I\kappa x)$

In addition, the categorical properties of ordinary things are associated with categorical properties of certain proper things. In particular, if Q is constituted by (x, κ), then x is a thing with Q. Every ordinary thing has just the categorical properties of some proper thing. And if z and Q are constituted by x and κ then necessarily z has just the categorical properties of a proper thing with Q.

O3 $\forall x \forall Q (\exists \kappa CQx\kappa \rightarrow IQx)$

O4 $\forall z \exists x (z \approx x)$

O5 $\forall z \forall Q [\exists x \exists \kappa (Czx\kappa \wedge CQx\kappa) \rightarrow \Box \forall w (IQw \leftrightarrow w \approx z)]$

Oe is a consequence of O4 and O5. And with O3, it follows that an ordinary physical thing constituted by some x and κ has all the same categorical properties as x, $\forall z \forall x (\exists \kappa Czx\kappa \rightarrow x \approx z)$ so that a statue and some clay with a common proper constituent have their categorical properties in common.

And, consistent with essentialism, there may be direct consequences from essence properties. In general, we expect that if Q is constituted by (x, κ), then anything with Q has κ.

O6 $\forall \kappa \forall Q [\exists x CQx\kappa \rightarrow \Box \forall w (IQw \rightarrow I\kappa w)]$

And there may be revised versions of principles like (P1) - (P3) on p. 66. Let P be BEING A PERSON,

P1′ $\forall_Q[\exists x\, C_Q x \text{P} \rightarrow \Box \forall y (I_Q y \rightarrow I_C y)]$

P2′ $\forall_Q[\exists x\, (C_Q x \text{P} \wedge I_F x) \rightarrow \Box \forall y (I_Q y \rightarrow I_F y)]$

P3′ $\forall_Q \forall_R[\exists w \exists x\, (C_Q w \text{P} \wedge C_R x \text{P} \wedge I_M x w) \rightarrow \Box \forall u (I_Q u \rightarrow \exists v (I_R v \wedge I_M v u))]$

If Q is constituted by (x, P), then necessarily if a thing has Q it has a certain sort of continuity; if Q is constituted by (x, P) and x is female, then necessarily if a thing has Q it is female; and if Q and R are constituted by (w, P) and (x, P) and x is the mother of w, then necessarily, if u has Q there is some v with R such that v is the mother of u. Once the consequents are detached by constitution and features of constituting objects, the result is the essentialist solution we expect. Suppose Sally is female and constituted by $(a, \text{BEING A PERSON})$, so that $C_s a \text{P}$ and $I_F s$; with O1 and O2, there is some Q such that $C_Q a \text{P}$; and with the result from O3, $I_F a$; so by P2′, $\Box \forall y (I_Q y \rightarrow I_F y)$; and from Oe, $\Box \forall w (I_Q w \leftrightarrow w \approx s)$; so with $\Box(s \approx s)$ by the indiscernibility of identicals, $\Box F s$. As before, grounds for these principles may be like those associated with the *de dicto* solution. O1 - O6, and so Oe are from BEING AN ORDINARY THING and P1′ - P3′ from BEING A PERSON. At the same time, from these principles, it remains that *individuals* have their modal properties only by virtue of features of their constituents.

Observe that features of constituents (a, K) do not translate directly into those of the ordinary thing they constitute. Proper constituents are a ground for ordinary things with their properties, and ordinary things are a function from the ground. But there is a gap between features of ordinary things and features of their grounds. So, for example, HAVING SUCH-AND-SUCH PARTS is an ordinary feature that results on the ground of a proper thing and kind property, but ordinary things do not have proper things and kind properties among their ordinary parts. And the statue and clay have modal features which are not themselves features of the constituting proper thing or kinds. Suppose there are some ordinary things with such-and-such features; they are constituted by proper things and kinds, with features of the ordinary things a function from the constituting base. From this function the ordinary things *supervene* in the sense of ST on the base of proper things and kinds. And we respond to Kim's challenge from "Supervenience as a Philosophical Concept" insofar as we do not assert mere covariance of ordinary things on an actual ground, but rather offer an account of the relation between them from the account of ordinary things (see p. 10).

Worries. This picture of ordinary things as supervenient on a ground of proper things and kinds may raise concerns about whether there "really" are such things. A free lunch may seem to be no luch at all. It is, however, difficult to get a grip on the question. Observe that the picture I offer is entirely objective. From the previous section, things proper together with kind properties result in *de re* modal features. The suggestion here is that ordinary things depend on that which is already fixed by proper things together with the properties. So the objects are not "conventional" — at least in the sense that claims about them somehow depend on us. Perhaps, then, the concern is that ordinary things are not concrete in the way that proper things are concrete. But ordinary things are concrete just insofar as the functions from proper things to ordinary ones result in things with particular qualities. So, for example, the statue and clay have some shape, mass, and the like. Like anyone else, I agree that one can stub one's toe on a rock.

Perhaps, then, the concern is that ordinary things cannot be anything *new*. This might be a question about functions from input to output objects. If the functions are conceived mathematically, as sets of pairs, the objects exist already as members of the functions — so there is nothing ontologically interesting about a functional relation on them. But I am thinking of functions more elementally: A function is an operation which takes some inputs and extrudes an output. We cannot quite think of a function as a sausage maker which takes some ingredients and extrudes a product, insofar as there is a *process* for sausage making. Perhaps (idealized) logic gates are a better analogy: A logic gate is a device to implement a Boolean function.
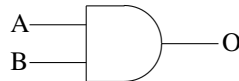
So for example when its inputs A and B both have the value (voltage) 1, the output O of an 'and' gate is 1, and otherwise the output is 0. The output "results" from the input — though the idealization abstracts from any process or mechanism by which this is so. For the output, both the inputs and the operation are required. And together they are sufficient.

Still, one might ask *what are* these ordinary things that are the outputs of functions? In general, the answer depends on the function. The statue has some matter (the same as of the proper thing that is the statue and the lump). And it has an identity property BEING THIS STATUE. Similarly, the lump has that matter but a different identity property. For an ordinary thing, the function from the base isolates its properties, including an identity property. After that, there is nothing more to be said about what it is like.

Perhaps this reply invites the objection that ordinary things fail to be distinct from their grounds in some required sense. Of course there may be nothing mysterious about one thing constituted by others. Bricks may constitute a wall distinct from them. But there are other examples: To borrow a familiar case, a physical computer is a *realization* of a Turing machine, where a realization is determinative of the properties it realizes, and physically constitutive of the individuals with those properties.[23] In this case, as in ours, the language of constitution indicates a distinction between properties of constituting objects at one level, and of a constituted object at another. But perhaps this example, with debates about supervenience and realization in philosophy of mind (as Kim, *Mind in a Physical World*) suggest that ordinary things somehow *reduce* to their grounds, and so do not exist as such. In the context of reduction, the mere observation that the properties which apply to ordinary things are not the same as properties of their ground does not suffice to show that ordinary things are distinct from their ground.

I have expended considerable effort to show that resources for the account of ordinary things are already contained in the non-essentialist view. So there is some force to this objection. Still, it is worth making some related points. First, the argument we have seen for nominalism against independent abstracta does not get its grip against ordinary things. Ordinary physical things are not abstract. But, further, ordinary things *depend* on ones with which we interact. And there may be a theoretical and top-down case for unreduced ordinary things. This case against reduction coincides with whatever reasons there are for thinking there exist objects of the different sorts. And the case may be considerably strengthened as we go on to consider ordinary things including events, fictional characters, sets and the like. From this perspective, the top down argument tells us that there are the things, and our account says how and what they are.

These considerations are related to another objection. Suppose there are ordinary things. Then there are a lot more ordinary things than one might have thought. The problem is not merely the "problem of the many" — that there are too many clouds where one has a drop of water that another does not, or too many cats where one has a single hair that the other does not (see, for example, Unger, "The Problem of the Many," and Lewis, "Many, But Almost One").[24] Not only this, but any given proper thing might have very many different kinds. There are BEING A STATUE and

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[23]This language reflects the "standard" view of realization from Wilson, "Two Views of Realization."

[24]I do not have a unique or developed reply to this problem. However, the difficulty is not unique to the present approach. And different responses might apply. So, for example, our account is compatible with one on which the probelem is one of vagueness or ambiguity (as McGee and McLaughlin, "The Lessons of the Many" and Weatherson, "Many Many Problems").

BEING SOME CLAY. And there are likely closely related properties in the neighborhood, BEING SOMEWHAT LUMPY CLAY, BEING CLAY A BIT MORE LUMPY THAN THAT, and the like (perhaps something like different grades of peanut butter) — with different ordinary things corresponding to each. So there are a lot of ordinary things! Short of developed accounts of ambiguity or vagueness, by whatever mechanism, some things rather than others are presumably ones we manage to talk about. Perhaps individuals from another culture, another planet, or galaxy would care to talk about different ones. But this is not fundamental to the metaphysical picture. And there need be no conflict with positive intuitions about the ordinary things there are; our view supplies a full complement, and lets individuals select among them. To the extent that there is room for conflict, it will likely be with intuitions about which things there are not. But I think these intuitions will be much more difficult to sustain.

It is by the explosion of things that we retain not only the structure of our solution to the statue-clay case, but also to the puzzle about the series of worlds, where lecterns in adjacent worlds differ by only a few molecules and so are the same, but lecterns in worlds at the endpoints are composed of completely different molecules and so are different. Though $a = b$, when when $\text{J} \neq \text{K}$, ordinary things composed by (a, J) and (b, K) are distinct. Thus the statue is not the same as the clay. Similarly, though $\text{J} = \text{K}$, when $a \neq b$, ordinary things composed by (a, J) and (b, K) are distinct. Say lecterns are ordinary things and the essence for this one is $\langle a, \text{BEING A LECTERN} \rangle$. Then some proper things in the series have this property, and so are categorically like this lectern in the different worlds. But consider another proper member of the series b, with the corresponding essence $\langle b, \text{BEING A LECTERN} \rangle$. This is a different property, and so corresponds to a different ordinary thing. Some members of the series might instantiate both these properties, so that the different lecterns would be co-located. This is odd — for many have claimed that distinct things of the same kind can never be in the same location. But observe that we get different lecterns in the same location only from an unintuitive starting point, with a and b located in different worlds. In the ordinary case, we are interested in things with a and b both actual; then if a and b share categorical properties, $a = b$ so that the thing constituted by $(a, \text{BEING A LECTERN})$ is the same as the one composed by $(b, \text{BEING A LECTERN})$. At any rate, as before, there may be questions of vagueness or the like, but there is no pressure to say the lectern at one endpoint is identical to the one at the other, and so no pressure to say they are both identical and distinct.

These are Quinean reasons to agree that there are ordinary things. However, the proposal is not merely that ordinary things fill a theoretical role, but also that consideration of what is in the world reveals that there are ordinary things. For the moment, set aside the details of our view, and begin with the question whether there

are things in arbitrary occupied spatiotemporal regions. It seems possible to identify spatiotemporal regions as sets of points and, subject to qualifications as before, ones that are occupied. This seems sufficient to identify the stuff in a region and assign it to a variable governed by an existential quantifier. And the existential quantifier indicates existence. More generally, there are different organizations of that which is in the world. Our theory accommodates arbitrary organizations. Given such an organization, the existential quantifier applies. Things exist when they are provided by the world on some organization, otherwise not. And this is no "thin" or "lightweight" existence. Consider some preferred range of things. Ordinary things exist like that. Denying that there are ordinary things is something like agreeing that there are an assembled frame, seat, wheels, handlebars, and the like, but denying a bicycle.

Of course some philosophers do just this! Thus they propose that there is a separate question, "when composition occurs." But, as Jubien observes, "this threatens to be a quasi-mystical way of viewing the matter. It's as if there's a very special, insensible phenomenon known as 'composition', which in some cases unites spatiotemporally separated objects into a single object but perhaps in other cases does not" (*Possibility*, 3). Against Jubien, one might argue that there is some class of essences which identifies the objects. But we have not been able to find a ground for selecting essences from among essence-candidates. Even nihilism and universalism may be theses about composition — roughly, in the one case that composition always fails, and in the other that it always obtains. Sider, *Writing the Book of the World*, argues that there are "joints" in the world fundamental for metaphysics where these matter for composition. Of course there are theoretically distinguished properties useful in a wide variety of analyses and applications — it matters which things are dogs, and which are not. The problem is how or why the question of such joints so much as arises for for (wide-open) application of the existential quantifier.

Suppose, then, that we set special criteria for composition and "thinghood" to the side. Then there seems no block to letting the quantifier range over that which occupies arbitrary spatiotemporal regions. And similarly, there is nothing to prevent quantification over domains including that which is constituted by arbitrary things proper and kind properties — or whatever. Our framework makes space for arbitrary arrangements of what there is. Given the way the existential quantifier works it applies to the ordinary things. So there are ordinary things. At least the observation that there are very many of them does not by itself defeat the proposal that the world has such things on offer.[25]

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[25]Of course there are *arguments* against multiple things in the same place at the same time and the like — as for example chapters 2 and 3 of Merricks, *Objects and Persons*. This discussion distracts from our main themes, though a complete defense would address such concerns one-by-one.

2.4.2 More Ordinary Things

Our picture is that ordinary physical things are composed by proper things with kind properties. But depending on grounds and functions, there may be ordinary things in addition to ones we have seen so far. The structure of functions from grounds to ordinary things supports a wide variety of objects: thus events, types and fictional characters may turn out to be no less ordinary than rocks, trees, statues, and clay. Physical things are ordinary. But they may be part of the ground for other ordinary things, which may be the ground for still more ordinary things, and so on. Thus, generalizing,

OT An *ordinary* thing t is constituted by a kind with some proper or ordinary thing(s) (aa, κ), where κ fixes a function from aa to t.

Again, functions are such that $\langle aa, \kappa \rangle$ is the essence of t. As we shall see, there is room for aa to be missing or empty. A proper a that is a constituent of an ordinary physical thing is world-bound. But the ordinary aa that are constituents of an ordinary t need not be so restricted. Still, it may be that actual features of aa contribute specially to constitution so that so that a thing is constituted by $(aa, \kappa)_w$. Though grounds and functions may differ from from grounds and functions for ordinary physical things, general mechanics do not. The case of ordinary physical things is, perhaps, particularly simple (!) and relatively well-worked-out. But it may be that theories of events, fictional characters and the like are precisely theories that would explain the supervenience of these things on a ground of things and kinds, and so account for them as ordinary things.

Though events such as a *walk* or *parade* are grounded on properties of an individual that takes a walk or individuals in a parade, such things do not take on all the properties of the ground individuals. So, for example, a walk does not have a *weight*, though the one who takes it does. Thus there are unique functions from grounds to things of the different kinds — and the structure leaves room for functions resulting in things that lack particular qualities altogether. Thus functions may be such as to result in abstract ordinary things. But this need not detract from their status as ordinary. So we recapitulate the case for ordinary physical things: Abstract or no, ordinary things remain objective. They are not "conventional" — at least in the sense that they depend on us. Similarly, ordinary things remain concrete to the extent that particular qualities are among their properties. But whether a thing has particular qualities seems irrelevant to whether the things are; as before, the ordinary thing is constituted by some ground thing(s) with the kind property; things are derived, though given the

ground, they exist in a perfectly robust sense. The cases of events, types and fictional characters illustrate the view.

Events. In "The Logical Form of Action Sentences," Donald Davidson gives a Quinean top-down argument for the existence of events. Starting from that (i) Jones buttered the toast in the bathroom with a knife at midnight, he observes the obvious consequence that (ii) Jones buttered the toast. But if we take (i) to express a single multi-place relation between, say, Jones, the toast, the bathroom, the knife and the time, and (ii) to express a distinct two place relation between Jones and the toast, the consequence does not fall out of standard predicate logic. $\exists v \exists w R^2 vw$ does not follow from $\exists v \exists w \exists x \exists y \exists z R^5 vwxyz$. Davidson restores the consequence by quantifying over the *buttering* as a thing, with modifiers introduced as predicates of it. So, $\exists x (x$ is a buttering of y by $z \wedge x$ was done in the bathroom $\wedge x$ was done with a knife $\wedge x$ was done at midnight). Then it follows that there is a buttering of y by z just by dropping conjuncts in the usual way.

So Davidson shows how to retain consequences we want to keep. And something of the sort seems required if we are to retain the consequence against the background of ordinary predicate logic. Davidson's reasoning is, perhaps, less than conclusive. Extraordinary logics may retain the consequence without the quantification over events. So, for example, in "Concerning the Logic of Predicate Modifiers," Romane Clark suggests a system on which new predicates are produced out of old ones by attaching *modifiers* to them. So long as the extension of *modP* is a subset of the extension of P it follows from $\exists x modPx$ that $\exists x Px$. So, where *x butters the toast with a knife* is a modification of *x butters the toast*, it follows from that Jones buttered the toast with a knife that he buttered the toast. And similarly for the more complex cases. Suffice it to say that this top-down semantic argument may be less than decisive. Still events may figure as relata for causes, as my *pulling the trigger* is the cause of *his dying*, and so may figure in theories for science and action. So there may remain top-down pressure in the direction of events. And we may look at the question from the other side, bottom-up, simply by asking what things there are from our account of things.

In a number of places, J. Kim has argued that events are just the having of properties or relations by things at times (see. for example, his, "Causation, Nomic Subsumption, and the Concept of Event" and "Events as Property Exemplifications"). Of course, there are other theories of events. Without plumping for his view, or requiring that it be right, his will do as a sample to suggest the *sort* of theory that one might offer for events as ordinary things. In the simplest case, on Kim's view, an event is a thing's having a property over some time, indicated $[x, P, t]$. Change the

thing, the property, or the time, and you change the event. So, depending on the time t, the event need not include the thing over its entire career, and depending on the property P (which may be complex), the event need not include all the properties of the thing. Kim's view makes room for very many events — his theory is sometimes criticised on the ground that there are too many. Thus there may be [Jones, WALKING, this afternoon], but also [Jones, WALKING LEISURELY, this afternoon] where, so long as WALKING is one thing and WALKING LEISURELY another, the events are distinct — though Kim would say one is *included* in the other. The result here is very much like the explosion of ordinary things on our view, and I am not inclined to criticise Kim's view on this ground. Perhaps we are only interested in certain of Kim's events.

Indeed, it seems natural to think that an event kind, perhaps, BEING A WALK selects just certain of a thing's properties, that the thing is walking, that it takes such-and-such route, and the like, so that from the thing and time (or the thing at the time) with the kind (x, K) is a function to an object of Kim's sort $[x, P, t]$, with properties appropriate to an event's being a walk. And similarly, some things at a time together with BEING A PARADE might single out certain properties of them, and similarly for some snow with BEING AN AVALANCHE, or a philosopher with CONTEMPLATING EVENTS. So far, the picture is as before: just as the statue is constituted by a proper thing with BEING A STATUE, or the clay by a proper thing with BEING SOME CLAY, so the event is constituted by person at a time with BEING A WALK.

But further, and perhaps in contrast to Kim, we arrive at a plausible account of essential properties as well. Suppose W is BEING A WALK, R is FOLLOWING SUCH-AND-SUCH ROUTE and an event e is constituted by $(a, \text{BEING A WALK})$ where the object a has R. Then $Cea\text{W}$ and $I\text{R}a$. By analogy with what we have seen before, we may expect principles for events on the order of,

E1 $\forall \text{Q}[\exists x(C\text{Q}x\text{W} \wedge I\text{R}x) \rightarrow \Box \forall y(I\text{Q}y \rightarrow I\text{R}y)]$

If Q is constituted by (x, W) and x follows a particular route, then necessarily any event that has Q follows (roughly) that route. Then from $Cea\text{W}$, with principles like O1 and O2 there is some Q such that $C\text{Q}a\text{W}$; so from E1, $\Box \forall y(I\text{Q}y \rightarrow I\text{R}y)$, and with Oe, $\Box \forall w(I\text{Q}w \leftrightarrow w \approx e)$; so with $\Box(e \approx e)$ by the indiscernibility of identicals, $\Box I\text{R}e$, and e necessarily follows (roughly) that route. Similarly, one might expect that an avalanche would be necessarily related to its causes, or a parade to its location or to certain of its floats. A particular parade might have started on-time instead of late, or might have had somewhat different entries; still, it is hard to see how my local Blue Jay Christmas Parade, which runs about a quarter of a mile (billed as the "World's Shortest Christmas Parade"), with its scouts and fire engines as typical entries, could be relocated to New York City, be sponsored by Macy's Department Store, and have

its entries replaced by giant floating balloons, but remain same parade. From the essence, we get what it is to be a particular κ, and so constraints on worlds in which the particular event may occur.[26]

So the proposal is that things with kinds result in a function to events. Properties of an event are not all the properties of the things in its ground. Rather, properties of an event are ones appropriate to its kind. And different event kinds could result in different properties and so different events on the same ground — as for example, my morning walk and morning exercise might be actually the same but modally different. Ordinary events may be concrete just insofar as event kinds result in properties which include particular qualities. Typically events have particular spatiotemporal locations among their properties. And they may have other particular qualities as well. Thus, if a parade is COLORFUL, and so includes particular color qualities, it will count as a concrete entity by our criterion. We get an *account* of the supervenience of these ordinary things by the theory of events, together with kinds as a map from grounds to the things. Of course, on different theories of the events, different accounts of the mapping are possible. But some such account of events as ordinary things is in the offing.

Types.   On the following line, there are two tokens of the syntactic type 'token',

<div align="center">token   token</div>

There is another token of the type in this sentence, and are multiple instances of it on this page and in this section. Types and tokens come in many varieties and at different levels. Thus the February 2007 game between the Chicago Bears and Indianapolis Colts was the XLI instance of the National Football League's Super Bowl. But the many NFL games leading up to that one were instances of the more general type, 'professional football game', and there is a yet more general type which includes as tokens the games played in the mud outside my house with a variable number of players and an iffy sort of ball.

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[26]In "The Individuation of Events" and other places, Davidson urges a view on which events are identical just in case their causes and effects are identical. This view has been criticized on a number of grounds, notably that it is circular: Insofar as causes and effects are events, $e = e'$ only if the causes of e are identical to the causes of e'; but these causes are identical only if their effects are identical, which is to say, just in case $e = e'$. Insofar as we allow that one event may be essentially linked to another, as an avalanche to its cause or to its effects, our view may seem subject to similar objections. But this is not so: nothing about the identity of the event, its BEING THE ONE IT IS depends on such relations. As for ordinary physical things, the things are interrelated, but do not require one another to be the ones that they are.

Let us focus on a type at an extremely low and simple level. Consider the lower-case, Times New Roman 'z'. Tokens of the type may differ in point-size, and have a particular color and location; instances are characterized by the particular ink or material of which they are made. In contrast, the type includes a shape capable of mathematical description.

As indicated by my font editor program, it is characterized by markers and outline as on the right.
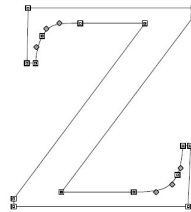
The outline scales according to point size, and maps to tokens of the type. To the extent that this is so, each token is of the *same* type. There is a distinction for types along the lines urged by van Inwagen for fictional characters. Types *have* some properties, but *hold* or *include* others — where a type may have the property of including whatever properties it does. Properties the type includes are ones that map to tokens. So the Times New Roman 'z' includes a shape but does not have it. It has the property of including the shape. Thus the type includes some, but not all of the properties of the tokens.[27]

Though it is sometimes mixed with mentalistic components, there is a traditional story about "abstraction" on which abstract entities include some but not all the properties of their instances. And it is natural to think that is what is happening here. For a type of a given kind, there is a class c of properties such that the type includes a property iff the property is in c and the token has it. Thus properties of the tokens are not included in the the type iff they are not in c. Given a different starting set c, there is a different resultant type — so the type for lower-case Times New Roman 'z' is one thing, and for the lower-case 11-point Times New Roman 'z' another, for the latter includes properties that the former does not, and so draws its instances from a smaller set than the former. In the case of types, it is natural to say that one is identical to another just in case it includes all the same properties as the other. Thus, if for any P, x includes P iff y includes P then then x and y are instances of the *same* type.

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[27]No doubt this case is too simple. So, for example, perhaps font types include relations to origin events, so that the Times New Roman 'z' would not be the same as a Times Old Barbarian character with the same shape but different origins. And there may be cases where origin events rather than instances are entirely constitutive, as a type of game designed but never played. But let us not worry about that for now.

An account along these lines puts us in a position to see types as ordinary things.[28] Given a thing, it may have types at many different levels. But a kind property selects some class c of properties such that from the thing and kind (a, κ) is a function to the type. In mathematics, there is nothing special about a many-one function on which different inputs have the same output. In this case, too, it may be that the output of (a, κ) is the same as that of (b, κ). Given the functional structure of our view, this seems a straightforward application. In the case of ordinary physical things, there is a one-one map from constituents $(\mathbb{a}, \kappa)$ to ordinary ordinary physical things. However the type constituted by (a, κ) is such that it might be the same as that constituted by (b, κ). In this case the type is *multiply* constituted.

Insofar as the identity of types depends on their included properties, perhaps it is natural to say types include properties necessarily. Say κ identifies a class including color properties. Then it may be natural to say,

T1 $\forall_Q[\exists x(C_Q x\kappa \wedge I_B x) \rightarrow \Box \forall y(I_Q y \rightarrow y$ includes $_B)]$

If $_Q$ is constituted by (x, κ), and x is blue, then necessarily anything that is $_Q$, includes BEING BLUE. So a physical object that might have been different with respect to properties in c, might have been of a different type than it is. Whatever we say about this, in form, this is very much like the account for ordinary physical things and events. The objectivity and reality of the thing is given by its ground and the function from the ground to properties of the supervenient thing.

Types have various properties, as being grounded in such-and-such way, and include many others. The constraint that different things, with different particular qualities, may result in the same type seems to guarantee that types do not include particular qualities. Insofar as a type includes just properties in c that are shared by individuals, and the individuals do not share their particular qualities, the type does not include their particular qualities. Similarly, types do not seem to have particular qualities. So the type is not concrete. So, in the usual case at least, types are ordinary but abstract. Thus we have a start at GP: there are some abstract things. Further, insofar as types are grounded in ordinary things and kinds, they have a concrete ground (so long as kinds themselves are so grounded); so we have at least the beginnings of the other part of GP according to which abstract objects are grounded on the concrete.

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[28]For contemporary versions of the traditional account see, Quine, "Logic and the Reification of Universals," who develops the point for a nominalistic conclusion, along with C. Wright, *Frege's Conception of Numbers as Objects*, and B. Hale, *Abstract Objects*, who move directly from language to platonistic results. As above, our proposal is rather that the structure fits already into an account of ordinary things.

Fictional Characters. We encountered some theories of fictional characters in chapter 1 (p. 16). Van Inwagen develops a theory motivated by top-down Quinean considerations. He makes a distinction between properties fictional characters *have* and properties they *hold*, though he refrains from saying all that much about the intrinsic nature of such things. Wolterstorff offers an account on which fictional characters are a species of eternal platonic entity. Thomasson allows that fictional characters are 'abstract artifacts' created by their authors. It is not my intent to engage this debate in the sense that I shall offer or defend an approach to fictional characters. However, I comment on one criticism van Inwagen urges against Thomasson: For our purposes, Thomasson's view is particularly interesting insofar as she makes it natural to treat fictional characters as a species of ordinary thing.

On Thomasson's account, "we should consider [fictional characters] to be entities that can come into existence only through the mental and physical acts of an author — as essentially created entities" (*Fiction in Metaphysics*, 6). According to Thomasson, fictional characters depend historically on the particular acts of an author or authors; they also depend continuously on the existence of some literary work(s) in which they appear. Similarly, a literary work depends on particular acts of its creator; it also depends continuously on the existence of some "copies" of an original product, as well as some individuals with the language capacieites and background assumptions to read and understand the work. Roughly, *x* and *y* are instances of the same *composition* if they are copies, related in an appropriate way to an original; *x* and *y* are the same *literary work* if they are instances of the same composition and also demand the same background assumptions and language capacities of readers. Then a fictional character is something ascribed properties in a literary work. Sufficient for the identity of characters *x* and *y* are that *x* and *y* appear in the same literary work and are ascribed all the same properties in that work. Necessary for the identity of characters *x* and *y* in works *K* and *L* is that the "author of of L must be competently acquainted with *x* of *K* and intend to import *x* into *L* as *y*." This condition is not sufficient for identity across works although, "if characters fulfill this condition, then we have good grounds for claiming that *x* is *y*" (67).

Thomasson distinguishs *real* contexts external to a story, and *fictional* contexts in which we say what is so according to a story. In real contexts, we attribute genuine properties to entities fictional or otherwise. So 'Jimmy Carter is a person', 'Hamlet is a fictional character', 'Julius Caesar appears in more than one literary work', and 'Hamlet appears in more than one literary work' are to be taken in the straightforward way as attributing properties to individuals. In fictional contexts we say what is so of characters fictional or otherwise according to stories. So 'Hamlet is a prince' and (in the context of Yalom's *When Nietzsche Wept*), 'Nietzsche was psychoanalyzed

by Freud' say what is so according to the stories (106-107). Thomasson argues that this account allows a uniform treatment of real and fictional objects, and a reasonable approach to characters that are attributed inconsistent properties according to stories, or appear in different works with different features (but see "Speaking of Fictional Characters," 210-14).

Thomasson catalogs dependence relations along a couple of different dimensions: things may be *rigidly* dependent on particulars, or *generically* dependent insofar as they require just that there are some object(s) of a given sort. Things may be *historically* dependent for their origin, and *constantly* dependent for continued existence. But she does not say much about the intrinsic nature of such dependence except that a necessary condition for the dependence of α on β is that "necessarily, if α exists, then β exists" (25). Similarly, she offers a system of ontological categories from the ways things depend on concrete and mental entities, and Quinean considerations for accepting objects in the different categories. Thus in *Fiction in Metaphysics*, Thomasson holds that there are fictional objects and that they covary in important ways with the actions of authors and the like. But beyond telling us that they are and stand in certain relations, she does not tell us what it is about fictional objects that establishes their relations with the other things. She does not tell us all that much more than van Inwagen about the nature of fictional objects.

But Thomasson's fictional characters appear naturally as ordinary things (we shall encounter Thomasson's own, more recent and rather different approach to ordinary objects in chapter 5). For a matter so complex, we are in a position to indicate only the outline of an application to Thomasson's theory. From considerations of the sort she proposes, a work of fiction is constituted by some text, context and a kind, BEING A WORK OF FICTION, (t, c, F). As for types, a work of fiction may be multiply constituted insofar as different instances of a text might result in the same work. Though contexts include mental states, on the account we have offered, works of fiction may remain objective insofar as they do not depend on mental states directed at those contexts. I suppose that among the features of works of fiction are that some sentences are true according to them (for example Lewis, "Truth in Fiction"). Then a fictional character is constituted by a work of fiction, a part or fragment of that work according to which something has a property, and a kind, BEING A FICTIONAL CHARACTER, (w, p, C). Fictional characters, too, are multiply constituted insofar as a given work with different fragments might constitute the same character, and a single character might be constituted from different works. If it is true in the work that the object from the fragment has some feature, then among the properties of the character constituted by (w, p, C) is BEING THAT WAY ACCORDING TO w. And fictional characters have straightforward features as well, as appearing in more than one work, or the like.

As Thomasson observes (109), modal claims have different readings. Thus 'Meursault could have refrained from killing the Arab' could be read as,

A. There is some story (*The Stranger*) such that according to it, it is possible that Meursault refrained from killing the Arab.

B. There is some story (*The Stranger*) such that, it is possible that, according to it, Meursault refrained from killing the Arab.

C. Possibly, there is some story such that, according to it, Meursault refrained from killing the Arab.

The first is, perhaps, the most natural. It depends on the modal features Meursault has according to the story. The second depends on modal features of the story as such, whether *The Stranger* could have been different. It is the last that depends on the modal features of the fictional character, whether it could appear in stories of a certain sort. This is a part of Thomasson's account that is incomplete. There is the necessary condition that the author of alternatives for Meursault be familiar with *The Stranger* and intend to write about him. But the condition is not extended to say whether, for example, there could be a Disneyfied version in which Meursault becomes a happy frog who would never hurt even a fly.

Though the account is far from complete, we have taken steps toward accounting for objects in Thomasson's theory. Thus, consider van Inwagen's comment, after discussing the relative virtues of Wolterstorff's and Thomasson's theories of fictional objects,
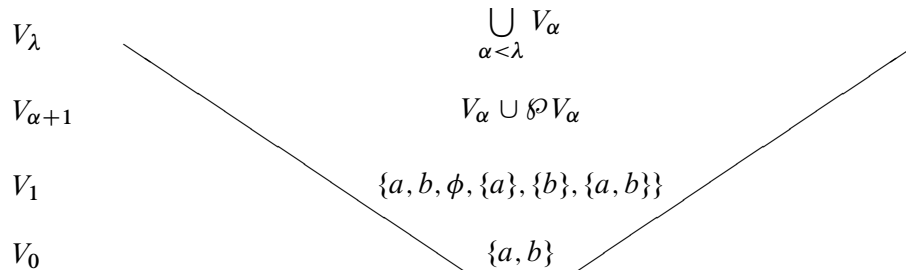
> Wolterstorff's theory is unintuitive in many respects (it cannot be reconciled with many of the things we are naturally inclined to believe about fictional entities), but it asks us to believe only in things that we, or the Platonists among us, were going to believe in anyway. Thomasson's theory respects what we are naturally inclined to believe about fictional entities, but it achieves its intuitive character by, as it were, brute force: by postulating objects that have the features we are naturally inclined to think fictional characters have. ("Fictional Entities," 155)

If her view requires no more than ordinary things for fictional characters, then van Inwagen is dead wrong — or, rather, only half right. We may agree that Thomasson's theory respects what we are naturally inclined to believe about fictional entities. But it requires precisely the ordinary things we were going to believe in anyway. Thomasson holds that fictional entities supervene on the activities of authors and the like. But this obtains on an account according to which fictional characters are ordinary things.

2.4.3 Towards a Formal Model

So far, proper things constitute ordinary things, which may constitute still further
ordinary things, and so forth. A natural thought is that things stack up in in a hierarchy
something like the iterative hierarchy of sets. I begin this section with consideration
of sets as ordinary things, and then turn to discussion of the general structure of the
universe of ordinary things.

Sets. On the standard account, there is an *iterative* hierarchy of sets. For "impure"
sets some atoms are allowed. Then at the lowest rank is a set V_0 of atoms. At the
next is the set V_1 which consists of all the members of V_0 together with all the subsets
of V_0 — that is, the union of V_0 and its powerset $V_0 \cup \wp V_0$. And, generally, any
$V_{\alpha+1} = V_\alpha \cup \wp V_\alpha$. Then given all the sets V_α of this sort, there is a limit set V_λ which
consists of the union of them all. And the process continues. So, $V_{\lambda+1} = V_\lambda \cup \wp V_\lambda$,
and so forth. Where the atoms are $\{a, b\}$, the result is an ever-expanding hierarchy as
follows.

V_λ $\qquad\qquad\qquad\qquad\qquad\qquad \bigcup\limits_{\alpha < \lambda} V_\alpha$

$V_{\alpha+1}$ $\qquad\qquad\qquad\qquad\qquad V_\alpha \cup \wp V_\alpha$

V_1 $\qquad\qquad\qquad\qquad \{a, b, \phi, \{a\}, \{b\}, \{a, b\}\}$

V_0 $\qquad\qquad\qquad\qquad\qquad \{a, b\}$

Something is a set just in case it has a rank, just in case it appears at some stage
in the hierarchy. A virtue of the iterative conception is that it explains or motivates
a certain resolution of the set-theoretic paradoxes. Thus, for example the Russell
paradox, which asks whether the set of all non self-membered sets is self-membered,
does not get off the ground. If all the sets are formed from ones at ranks before no set
is self-membered, and so all sets are non self-membered. And insofar as each stage is
followed by another there is no set of all sets, and so no set of all non self-membered
sets.

   The most natural interpretation of the iterative hierarchy is platonic — so that
the entire structure has an objective and abstract existence. There are top-down
motivations from the utility of mathematics (at least up to certain ranks). To the extent
that one is worried about such motivations, though, skepticism about abstract objects
in general or about sets in particular may attach to the hierarchy (especially at stages
after ones with Quinean motivation). In addition, it has proven difficult to say just

what relates sets at levels of the iterative hierarchy. "Formation" is at best a metaphor if formation comes to some sort of process. No process located in (some sort of) time will ever run through all the sets. A natural thought is that sets depend upon their members in the sense that sets at higher rank cannot exist apart from their members. But it is usually thought that ϕ and $\{\phi\}$ exist in every world; and just as there is no world with {Quine} without Quine, so there is no world with Quine without {Quine}. So sets at lower rank cannot exist apart from ones at ranks above — and modal notions do not obviously capture the required dependence of sets above on ones below.[29] In one place Boolos suggests a structural picture according to which sets are related, or may be listed, in ranks apart from any notions of formation or dependence ("Iteration Again," 90-91). But this seems to evacuate the iterative picture of its motivational force. The iterative picture is meant to tell us not only that there are certain sets, but why there are the sets there are, and so why the sets do not run into paradox.

Where sets are ordinary things, however, there is a response to such concerns. Thus, with s the kind BEING A SET, a set $\{aa\}$ is an ordinary thing constituted by (aa, s). The function from constituents to the set may result in the standard features, and in particular that identity depends just upon members. In this case, the function set up by s is multigrade, allowing any number of arguments so that the kind operates on any objects to form a set. And given some sets as ordinary things, there are further sets constituted of them, and so a hierarchy of sets; we offer an account of the relation between sets in the iterative hierarchy insofar as sets at one level are constituted by ones before. Where the mereological combination $((a, b), (a, b))$ is just (a, b), the set $\{\{a, b\}, \{a, b\}\} \neq \{a, b\}$. This is why there is no direct move from mereology to sets (recall our discussion of pairs from p. 32). However, with $(a, b)^*$ the object constituted by (a, b), there is a difference between $((a, s)^*, (a, s)^*, s)^*$ and (a, s) insofar as the one results by a function operating on the results of functions with final output $\{\{a\}, \{a\}\}$, and the other is just some constituents that are not a set at all. So it is important that constituted objects have their own identity, distinct from constituents.

One might worry that the empty set poses a special problem for the current proposal. Where ordinary things are constituted by kinds with some input objects, there is apparently no output apart from the input objects. However, there is room to understand constitution so that it operates even on an empty input. Return to our elemental notion of a function as a sort of "black box" taking some input(s) and extruding an output. In this case,

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[29]Parsons, "What is the Iterative Conception of Set?" considers and rejects different process approaches to formation. He advocates a modal account.
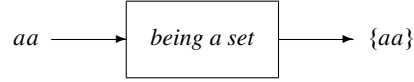
Then it seems natural to think that, as for a physical device of this sort, the operation could have a "default value" in the absence of any input. In this case, then, $(-, s)^*$ is ϕ.

So the proposal is that grounding for ordinary things underlies the iterative conception of sets. Correspondingly the iterative structure of sets illuminates the grounding relation for ordinary things. In "Iteration Again," Boolos offers a theory of the iterative conception usefully applied to flesh out a picture of sets as ordinary things. On this account, there are variables x, y, z for sets and r, s, t for stages. There is a stage-stage relation $s < t$ read 's is earlier than t' and a set-stage relation xFs read 'x is formed at s' (a set is formed at any stage after a stage where it is first formed); $x \in y$ is read in the usual way. yBs abbreviates $\exists t(t < s \wedge yFt)$ and is read, 'y is formed before s'. There are six axioms and one axiom schema.[30] These axioms fall into three groups. First, concerning levels: $<$ is transitive; $<$ is connected; each level precedes another; and there is a limit stage — a stage later than some stage, but such that no stage immediately preceeds it.

(*tra*) $r < s \wedge s < t \rightarrow r < t$

(*con*) $s < t \vee s = t \vee t < s$

(*pre*) $\exists r(s < r)$

(*inf*) $\exists r[\exists t(t < r) \wedge \forall t(t < r \rightarrow \exists s(t < s \wedge s < r))]$

For the last, there is a stage r with stages earlier than it; and any t earlier than r has an s greater than t but still earlier than r. There is a pair of axioms to regulate where sets are formed: all sets are formed at some stage; and a set is formed at a stage iff all its members are formed at earlier stages.

(*all*) $\exists s(xFs)$

(*when*) $xFs \leftrightarrow \forall y(y \in x \rightarrow yBs)$

Finally there is an axiom schema according to which if some things are all formed prior to a stage s, then there is a set of all those things.

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[30]I adopt a minor modification relative to the presentation in Boolos. Boolos observes that this option implies his own.

(*spec*) $\exists s \forall y [\mathcal{P}(y) \rightarrow yBs] \rightarrow \exists z \forall y [y \in z \leftrightarrow \mathcal{P}(y)]$

If there is a stage such that everything that is $\mathcal{P}$ is formed before it, then there is a z such that something is a member of z iff it is $\mathcal{P}$. The schema identifies things formed prior to stage s by a formula $\mathcal{P}(y)$. But there are only countably many formulas and so the schema applies to only countably many collections. However the principle has a (second-order or) plural motivation without this expressive limitation, $\forall xx (\exists s \forall y [y \prec xx \rightarrow yBs] \rightarrow \exists z \forall y [y \in z \leftrightarrow y \prec xx])$; for any xx if anything among them is formed before some s, then there is a z such that y is a member of z iff y is among the xx.

From these principles, Boolos derives standard axioms of set theory including empty set, pairing, union, power set, separation, infinity and foundation. So, for example, union: for any set a there is a set x whose members are members of the members of a.

> $\exists x \forall y [y \in x \leftrightarrow \exists z (z \in a \wedge y \in z)]$. For an application of (*spec*) let $\mathcal{P}(y)$ be $\exists z (z \in a \wedge y \in z)$; we show $\exists s \forall y [\mathcal{P}(y) \rightarrow yBs]$ and apply *spec*. By a quantifier placement rule, $\exists s \forall y [\exists z (z \in a \wedge y \in z) \rightarrow yBs]$ is equivalent to $\exists s \forall y \forall z [(z \in a \wedge y \in z) \rightarrow yBs]$. By (*all*), for some s, aFs. Now suppose $z \in a$ and $y \in z$; from aFs, by (*when*) $z \in a \rightarrow zBs$; so zBs and from the definition of B there is an $r < s$ such that zFr; from zFr and (*when*) again, $y \in z \rightarrow yBr$; so yBr and there is a $q < r$ such that yFq; by (*tra*) $q < s$; so yBs. So $z \in a \wedge y \in z \rightarrow yBs$; generalizing, $\exists s \forall y [\exists z (z \in a \wedge y \in z) \rightarrow yBs]$; so by (*spec*) $\exists x \forall y [y \in x \leftrightarrow \exists z (z \in a \wedge y \in z)]$.

Where a is formed at stage s, anything that is a member of a member of a is formed at a stage before s; so by (*spec*) there is a set of the members of the members of a. Thus, understanding the axiom of extensionality as a presupposition for any theory of sets, Boolos arrives at all the axioms of Zermelo set theory, that is all the axioms of ZFC less the axioms of replacement and choice. A theory of levels of the sort we have outlined is sufficient for a theory of sets sufficient to recapitulate a good part of classical mathematics (as, for example Potter, *Set Theory and its Philosophy*). And, supposing that Boolos's iterative picture falls out of and illuminates the account of ordinary things, so much is part of the account of ordinary things as well.

Boolos is not so sanguine about the axioms of choice and replacement. Perhaps, though, there is room for some tentative positive remarks. Boolos is, of course, correct that choice and replacement do not follow from the stated theory of the iterative conception. On his account, the iterative conception yields a structure. But beyond the assertion of a limit level and the countably sets from (*spec*) it does not do a good

job saying what sets there are (unless, of course, those are all the sets there are). So far, (*spec*) is an iteratively limited constructive principle. And the (second order and) plural version of *spec* does no better without a prior account of the sets upon which it operates. But we have seen the iterative conception driven by an account of sets as ordinary things. From this account, together with the plural version of (*spec*), there remains something to say about choice and replacement.

According to the axiom of choice, for every set a of non-empty disjoint sets there is a set c containing exactly one element from each member of a. The axiom is implausible if one requires specification or construction of sets. But it may seem natural in the current context — given the members, of course there is a set of them all. The axiom is trivial in case a has no members; so suppose a is non-empty. Consider some non-empty set a and suppose the antecedent of the axiom: any member of a is non-empty, for any $x \in a$ there is some $y \in x$; and members of a are disjoint, if x and y are different members of a, no z is a member of both.

(i) $\forall x[x \in a \rightarrow \exists y(y \in x)]$

(ii) $\forall x \forall y[x \in a \wedge y \in a \wedge x \neq y \rightarrow \neg \exists z(z \in x \wedge z \in y)]$

Where a is non-empty, there is some $x \in a$ and by (i) there is some $y \in x$; so $\exists y \exists x(y \in x \wedge x \in a)$; thus there are some objects xx such that a thing is among them iff it is a member of a member of a, which is to say (iii) $\forall y[y \prec xx \leftrightarrow \exists x(y \in x \wedge x \in a)]$.[31] By (iii) from left-to-right, if $y \prec xx$ then there is some $x \in a$ such that $y \in x$. And from (i) - (iii) together, for any $x \in a$, there is a $y \in x$ such that $y \prec xx$ and y is a member of no other member of a.[32]

A. (1) $\forall y[y \prec xx \rightarrow \exists x(y \in x \wedge x \in a)]$

(2) $\forall x[x \in a \rightarrow \exists y(y \in x \wedge y \prec xx \wedge \forall z(z \in a \wedge y \in z \rightarrow z = x))]$

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[31]The comprehension principle is of the sort, $\exists y \mathcal{P}(y) \rightarrow \exists xx \forall y[y \prec xx \leftrightarrow \mathcal{P}(y)]$. In one place Boolos suggests that any English instance of this form "expresses a *logical* truth if any sentence of English does" ("Reading the *Begreffsschrift*," 167).

[32]Suppose (i), (ii), (iii) and for conditional introduction that some $j \in a$; then with (i) there is some $y \in j$; so $\exists x(y \in x \wedge x \in a)$; so with (iii), $y \prec xx$. Again for conditional introduction suppose some $k \in a \wedge y \in k$; and for negation elimination $k \neq j$; then $k \in a \wedge j \in a \wedge k \neq j$; so with (ii), $\neg \exists z(z \in k \wedge z \in j)$; so by a quantifier rule, $\forall z \neg(z \in k \wedge z \in j)$; so $\neg(y \in k \wedge y \in j)$; but $y \in k \wedge y \in j$; so $k = j$; so $k \in a \wedge y \in k \rightarrow k = j$; generalizing, $\forall z(z \in a \wedge y \in z \rightarrow z = j)$; so that $y \in j \wedge y \prec xx \wedge \forall z(z \in a \wedge y \in z \rightarrow z = j)$ and $\exists y(y \in j \wedge y \prec xx \wedge \forall z(z \in a \wedge y \in z \rightarrow z = j))$; so $j \in a \rightarrow \exists y(y \in j \wedge y \prec xx \wedge \forall z(z \in a \wedge y \in z \rightarrow z = j))$; and generalizing again, $\forall x[x \in a \rightarrow \exists y(y \in x \wedge y \prec xx \wedge \forall z(z \in a \wedge y \in z \rightarrow z = x))]$.

Intuitively, since the xx are the members of members of a, for each $x \in a$ conditions on the axiom of choice guarantee at least one $y \in x$ such that $y \prec xx$ and y is in no other member of a.

To obtain the axiom of choice, we require not the xx which are all the members of members of a, but some yy from among the xx that include just one member from each of the members of a — that is, that there are yy such that $\forall y(y \prec yy \to y \prec xx)$ and,

B. (1) $\forall y[y \prec yy \to \exists x(y \in x \land x \in a)]$

(2) $\forall x[x \in a \to \exists y(y \in x \land y \prec yy)]$

(3) $\forall x \forall y \forall z[x \in a \land y \prec yy \land z \prec yy \land y \in x \land z \in x \to y = z]$

If $y \prec yy$ then y is a member of a member of a; if x is a member of a, then some member of it is among yy; and if x is a member of a, and y and z are members of x among yy, then $y = z$. Given that the yy are among xx, (1) is immediate from its counterpart in (A). B.2 and B.3 are like A.2 except that they require that the yy include one and only one y from each $x \in a$. From these, there is a direct line to the axiom of choice: By (*all*) there is some stage s such that aFs. Suppose $y \prec yy$; by (1) there is some x such that $y \in x$ and $x \in a$ and, reasoning as for union, by (*when*) and (*tra*), yBs. So $y \prec yy \to yBs$ and, generalizing, $\exists s \forall y(y \prec yy \to yBs)$. So by the plural version of (*spec*), $\exists c \forall y[y \in c \leftrightarrow y \prec yy]$. Then with (2) and (3), which tell us that there is exactly one $y \prec yy$ from each member of a, it follows that for any $x \in a$ there is a y such that $y \in c$ and $y \in x$ and no other member of c is a member of x.

$$\exists c \forall x[x \in a \to \exists y[y \in c \land y \in x \land \forall z(z \in c \land z \in x \to y = z)]]$$

And this is the consequent of the axiom of choice.

The key to this reasoning is the move from (A) to (B). Boolos is not inclined to deny the axiom of choice. Rather, having proposed a principle like (B), he suggests that although (B) may be apparent, "a sceptic about choice would immediately be skeptical about the truth of (B); one inclined to think that there need not be a set having exactly one member in common with each member of a would hardly suppose that there need be any such sets as are claimed to exist in (B). (B) may be perfectly obvious, but it is not *the iterative conception* that shows (B), or choice to hold" (97, constant symbols modified). One way of objecting to the axiom of choice combines a constructive approach to sets with the observation that choice, unlike the other axioms, asserts existence without the specification of a unique object asserted to exist. Thus,

for example, union specifies a unique set that is the union of a, so that it and other axioms construct or specify sets asserted to exist in a way that choice does not. Thus (B) asserts the existence of yy apart from a specification of them. Still, one might think that a realistic version of the iterative conception requires that if there are the xx of (A), then there are the ones of them that are the yy from (B). It is perhaps not the iterative conception that shows choice to hold, but the iterative conception together with a certain picture of the operation by which sets are formed. And we get a realistic version of the iterative conception from the account of sets as ordinary things: Any of the xx and so all of the yy exist as ordinary things upon which BEING A SET may operate. The result of this operation, by the (plural) separation principle, is the choice set. Taken together, these steps suggest that a motivation for accepting that choice is embedded in, or comes together with, sets as ordinary things.

According to the replacement axiom scheme, whenever a formula defines a function from the members of set a onto some objects, there is a set of those objects.

$$\forall x \forall y \forall z [\mathcal{P}(x, y) \wedge \mathcal{P}(x, z) \rightarrow y = z] \rightarrow \exists x \forall y [y \in x \leftrightarrow \exists z (z \in a \wedge \mathcal{P}(z, y))]$$

If $\mathcal{P}(x, y)$ is functional, then there is a set that has a member y just in case $\mathcal{P}(z, y)$ for some $z \in a$. From the axioms of Zermelo set theory (with or without the axiom of choice) there is an empty set $\phi = 0$; there is $\phi \cup \{\phi\} = 1$ and in general there there is $n + 1 = n \cup \{n\}$. Then, given the axiom of infinity, there is the set ω of all the integers. Then there is $\omega \cup \{\omega\} = \omega + 1$ and in general, where $f(0) = \omega$ and $f(n + 1) = f(n) \cup \{f(n)\}$, there is $\omega + n = f(n)$. But Zermelo set theory does not imply that there is a set with each $f(n)$ as a member, and so that there is $\omega + \omega$. Boolos conceeds that his stage theory so far formalizes "only a part of the iterative conception" (97). With the axiom of replacement, however, given the set ω there is the set of objects related by f to the members of ω that is, $\{f(n) \mid n \in \omega\}$, and the union of this set is $\omega + \omega$. And there are (much) larger sets too.

Intuitively, replacement is a "derived collection" principle. A collection of all the members of a together with a function from the members of a to some objects itself collects those objects and so demonstrates the existence of a set of them. To the extent that it is plausible, this is a constructive motivation, rather than a motivation from the iterative conception of sets. There is also a point about size — if a collection has no more members than some set a, then the collection must itself be a set. Motivations from the iterative conception tend to be as follows, "suppose that certain levels are correlated with the members of a set. Then that collection of levels can be considered as completed, in the sense that there must be a further level beyond (or above) all of them." Then if each $x \in a$ is correlated with the level of the set y such that $\mathcal{P}(x, y)$, there is a level beyond all of them, at which $\{y \mid \exists z (z \in a \wedge \mathcal{P}(z, y))\}$ must exist

(Drake and Singh, *Intermediate Set Theory*, 32; compare Shoenfield, *Mathematical Logic*, 239-240). As a justification for replacement, this reasoning apparently requires application of a derived collection principle to levels. At best, then, this reasoning fails to supply an independent motivation for replacement from the iterative conception of sets.

On our account, the iterative conception of sets characterizes a constitution operation by which there are the sets at one stage given the ones at stages below. In demonstration of the foundation axiom, Boolos establishes a least-member principle for stages, so that stages are well ordered by $<$, and any collection of stages has a least member. Given this, the constitution operation is sufficient for the sets at every stage: Suppose otherwise; then there is some stage at which the constitution operation does not result in the sets; since the stages are well-ordered, there must be a least stage s at which this is so; so the constitution function results in the sets at stages prior to s; and given these sets, the constitution operation results in the sets at s. So the constitution function results in the sets at every stage. This reasoning applies to any well ordered sequence of stages. Corresponding to the point about Zermelo set theory, it follows from the theory of stages that there are levels $0, 1 \ldots \omega, \omega + 1 \ldots$ but it does not follow that there is a stage $\omega + \omega$.

But the mechanism of constitution is sufficient to operate at a stage in any well-ordered sequence of stages. Thus it is natural to accept a *restricted* stage ordering or collection principle:

RC If any set a is well ordered, then there are stages ordered isomorphically to the members of it.

The idea is that sets are collected according to any well-ordered sequence of stages. So given a well-ordering, there are stages corresponding to it. And given further well-orderings, there are stages corresponding to them as well. Insofar as stages remain well ordered, sets are constituted at every stage in the hierarchy. Again, such considerations are not built into the bare iterative conception, but seem required on the iterative conception together with an operation of set formation whose application is to any well-ordered sequence of stages — so that set formation according to restricted stage collection is included or required with the account of sets as ordinary things. Insofar as sets of order-type $\omega + \omega$ appear in the hierarchy prior to that stage, this is sufficient to introduce levels $\omega + \omega$ and beyond.[33]

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[33]Given the countable infinity of objects (along with pairs of them) at stage ω, at stage $\omega + 1$ there is a set a and relation on a, as $0, 2, 4, \ldots 1, 3, 5, \ldots$ where evens and odds are ordered in the usual way but every even is less than any odd, such that its members are ordered isomorphically to

Even so, restricted collection is not sufficient for replacement. Still, apart from (*inf*), the iterative theory does not by itself require even the first infinite stage ω. In effect, then, replacement is another postulate according to which some stages exist. We are working with a picture on which there are arbitrary applications of the function from BEING A SET. Given this, so long as an iterative structure can extend to include some stage, it is natural to think that the function operates so that the structure does extend to include that stage. Supposing the consistency of ZFC set theory, replacement is consistent with the iterative conception. So we are positioned to accept replacement (and axioms stronger still). These axioms outrun what we were able to find just by the set function as applied to sets from the stated iterative conception. Still, the arbitrary nature of the operation gives reason to accept all of ZFC, and maybe more.[34]

So there are sets, and sets appear as a species of ordinary thing. Ordinary things in general and sets in particular exist according to principles compatible with the iterative conception of set and so at least Zermelo set theory. This account is sufficient for much of classical mathematics. Plausibly, sets are constituted compatibly with the axiom of choice, restricted collection and, more tentatively, replacement.

A Model. Sets are in an iterative hierarchy. And ordinary things more generally appear in a structure like the iterative hierarchy of sets. With dots for proper things and boxes for ordinary ones built of them, the picture is roughly as follows.
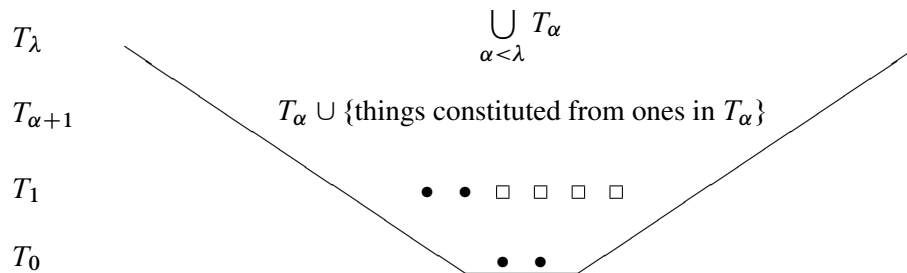
Given the kinds, at the bottom are proper things. Next are the proper things together with ordinary things constituted by them. And, in general, ordinary things at level

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$0, 1 \ldots, \omega, \omega + 1, \ldots$; and a stage with prior stages isomorphic to the members of this ordering would be stage $\omega + \omega$.

[34]Parsons, "What is the Iterative Conception of Set?" 513n14, suggests restricted stage collection and observes that it insufficient for replacement. Insufficiency is proved by Montague, "Semantical Closure." Potter, *Set Theory and its Philosophy* (218) considers a principle like restricted collection as a supplement to his basic system, and describes some features of the result. Boolos trains a sort of "incredulous stare" on both the full ZFC, and so replacement, as well as restricted collection, "Must We Believe in Set Theory?" 127.

$\alpha + 1$ are ones from α, together with ones constituted from them. A limit level collects things constituted this way.

It is natural to think a structure of this sort would have an image or model on the sets. For this, first, in the mathematical case, an n-place function applies to a sequence of objects $\langle a_1 \ldots a_n \rangle$. On our account a work of fiction is constituted by a kind applied to a text and context taken in order (t, c, F), and a set is ordered by a kind with some unordered things, (aa, S). Thus we picture ordinary things as constituted by partially ordered sets and kinds $\langle a, \text{R}, \text{K} \rangle$ where a is a set of objects, R is a (possibly empty) relation on a, and K is a kind. Second, different triples may constitute the same ordinary thing. This may occur if the same objects constitute a thing under different orders. But, further, as in the case of types, different objects may constitute the same thing. Thus we model ordinary things as sets of triples any of which would constitute the same thing.

For this, begin with sets t of proper things and k of kinds. Then working from the bottom up,

$$T_\lambda = \bigcup_{\alpha < \lambda} T_\alpha$$

$$T_{\alpha+1} = T_\alpha \cup T'_{\alpha+1}$$

$$T_0 = t$$

At the bottom are proper things. Members of $T'_{\alpha+1}$ are sets of triples $\langle a, \text{R}, \text{K} \rangle$ where $a \subseteq T_\alpha$, R is a relation on a, and k is a kind. Some sets of triples are not of the right sort to represent an ordinary thing. So, if both $\langle a, \text{R}, \text{K} \rangle$ and $\langle a', \text{R}', \text{K}' \rangle$ are in a member of $T_{\alpha+1}$ then $\text{K} = \text{K}'$. More generally, any triple in a member of $T_{\alpha+1}$ corresponds to an ordinary thing; and different triples in a member of $T_{\alpha+1}$ correspond to the same thing. Limit levels collect objects from levels before. So for example, $\{\langle \{a, b\}, \phi, \text{S} \rangle\}$ — the unordered set $\{a, b\}$ with BEING A SET, is the image of the set $\{a, b\}$; given such sets, and ones with them as constituents, all the sets appear in the hierarchy of ordinary things. Perhaps a partially ordered set which takes a text and then some elements of a context, together with BEING A WORK OF FICTION as $\{\langle \{t, c_1, c_2\}, \{\langle t, c_1 \rangle, \langle t, c_2 \rangle\}, \text{F} \rangle\}$ is a work of fiction. And a type may be of the sort, $\{\langle \{a\}, \phi, \text{K} \rangle, \langle \{b\}, \phi, \text{K} \rangle\}$. Thus an image of the ordinary things is in the hierarchy of sets.

Here is a sort of objection to this iterative picture from Johnston, "Hylomorphism." Suppose a group, the Australian Council of Trade Unions (ACTU) has as members such groups as the Fitters' and Turners' Union and the Dock Laborers' Union. Associated with the "principle of unity" for the ACTU is a "function from the constitutive unions to the ACTU. This function is plausibly taken to be item-generating, for the

ACTU seems ontologically dependent on its constitutive unions" (677). So far, so good. But suppose the ACTU itself authors another entity, the Bank Tellers' Union which then becomes a member of the ACTU. Insofar as the origin of the Bank Tellers' Union constitutively involves the ACTU, "the ACTU is not ontologically dependent on its consitutive unions" (678). Say this is right. Still, we may distinguish different versions of the ACTU (or the ACTU at different times) and hold that the one is among the constituents of the other. Suppose the Fitters' and Turners' Union and Dock Laborers' Union emerge at some level of the hierarchy q; then the initial version of the ACTU which has them as members is at some $r > q$; and, given the origin story, the Bank Tellers' Union is at some $s > r$; finally the ACTU with the Bank Tellers' Union as a member is at a $t > s$. So there is nothing about the story to suggest that the ACTU does not appear at some level(s) of the hierarchy. I agree, however, that among generative constituents of the ACTU is the ACTU at a different time. So the ACTU is not *merely* dependent on its member unions. This point will matter below.

Property Constituents. Suppose you conceive of Mark Twain with white hair and a walrus mustache. Your mental representation does not itself have white hair or a mustache, though it does somehow involve — or, one might say, "encode" such properties. In different places, E. Zalta advocates a theory on which there are concrete objects like Twain which have or exemplify properties, along with abstract objects which not only exemplify properties, but *encode* them as well.[35] On this view, for any expressible condition on properties, there is an abstract object that encodes properties meeting that condition; abstract objects are identical iff they encode the same properties. This theory has been presented as having Meinongian roots. So one encounters paradoxical sounding claims according to which there are things, the abstract objects, that do not exist. However, as Zalta demonstrates in his first book, his theory may be reconceived on a version according to which everything exists, but some existing things are the abstract objects (*Abstract Objects*, 51). Then the theory has a straightforward top-down Quinean motivation. And Zalta draws on his general theory for accounts of possible worlds, fictional characters, and the like.

Whatever one thinks of Zalta's analyses, if there are properties, they might count as constituents of things. So perhaps (PP, A) compose a thing where PP are some properties and A is BEING A (ZALTA-STYLE) ABSTRACT OBJECT. And properties might appear as constituents of things of other kinds — perhaps some types are constituted directly this way. It is perhaps natural to accommodate this by somehow including

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[35]See especially Zalta, *Abstract Objects* and *Intensional Logic*. The view is closely related to T. Parsons, *Nonexistent Objects*.

properties at the bottom of our structure among entities that constitute things, and continuing as before. But as Zalta himself observes, his theory has the strange consequence that some distinct abstract objects have all their properties in common. On our simple picture we can see this from cardinality considerations: With properties at the bottom of the structure, things at the next levels are like the powerset of properties; so there are more things than properties, and there must be things with all the same properties. The situation for Zalta's theory is more complex, but the result is the same.[36]

And the result matters. So, for example, in "Natural Numbers and Natural Cardinals as Abstract Objects," Zalta locates a theory of natural cardinals and natural numbers in the more general theory of abstract objects. This theory quantifies over abstract objects, but does not count them. This is because there is no one-to-one correspondence on objects that have all their properties in common: if a and b have all their properties in common and a has STANDING IN RELATION R TO OBJECT C then b has STANDING IN RELATION R TO OBJECT C; so R is not one-one. Similarly, Linsky and Zalta emphasize that abstract objects are individuated by properties they encode, and that having been so individuated abstract objects may contingently exemplify properties BEING THOUGHT ABOUT BY X, BEING STUDIED BY Y, INSPIRING Z TO ACTION, and so forth ("Naturalized Platonism," 538.) They say, "it follows from the comprehension and identity principles for abstract objects that for every condition on properties there is a *unique* abstract object that encodes just the properties satisfying the condition. . . . There is, therefore, a straightforward account of reference to abstract objects" (546). And "knowledge of particular abstract objects does not require any causal connection

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[36]Zalta develops a model for his theory on which objects outnumber properties (as "Natural Numbers and Natural Cardinals as Abstract Objects," 625-628). So his theory is consistent with the suggestion that there are more objects than properties. However the theory does not imply that there are more objects than properties. Rather the theory yields abstract objects with their properties in common as follows: Set Fx when x exemplifies F, and xF when x encodes F; an abstract object encodes F for any condition on properties, $\exists x[A!x \land \forall F(xF \leftrightarrow \mathcal{P})]$; and abstract objects are identical iff they encode the same properties, $x = y \leftrightarrow \forall F(xF \leftrightarrow yF)$. From these, there is a unique abstract individual k, that encodes F iff there is a y such that y does not encode F and F is [being a z with all its properties in common with y], $\forall F(kF \leftrightarrow \exists y(\neg yF \land F = [\lambda z \forall G(Gz \leftrightarrow Gy)]))$. Ask whether k encodes [being a z with all its properties in common with k]. Suppose not; from $\neg k[\lambda z \forall G(Gz \leftrightarrow Gk)]$, with =I, $\neg k[\lambda z \forall G(Gz \leftrightarrow Gk)] \land [\lambda z \forall G(Gz \leftrightarrow Gk)] = [\lambda z \forall G(Gz \leftrightarrow Gk)]$; so $\exists y\{\neg y[\lambda z \forall G(Gz \leftrightarrow Gk)] \land [\lambda z \forall G(Gz \leftrightarrow Gk)] = [\lambda z \forall G(Gz \leftrightarrow Gy)]\}$; so from the specification of k, k does encode $[\lambda z \forall G(Gz \leftrightarrow Gk)]$; reject the assumption: k encodes $[\lambda z \forall G(Gz \leftrightarrow Gk)]$. So from the specification of k, there is an object l such that $\neg l[\lambda z \forall G(Gz \leftrightarrow Gk)] \land [\lambda z \forall G(Gz \leftrightarrow Gk)] = [\lambda z \forall G(Gz \leftrightarrow Gl)]$; since k encodes $[\lambda z \forall G(Gz \leftrightarrow Gk)]$ and l does not, $k \neq l$. But from λ-conversion (what is obvious) k is an object with all its properties in common with k, $[\lambda z \forall G(Gz \leftrightarrow Gk)]k$; and since $[\lambda z \forall G(Gz \leftrightarrow Gl)]$ is the same property, $[\lambda z \forall G(Gz \leftrightarrow Gl)]k$. So there is an $l \neq k$ such that k has all its properties in common with l.

to them, but we know them on a one-to-one basis because de re knowledge of abstracta is by description. All one has to do to become so acquainted de re with an abstract object is to understand its descriptive, defining condition, for the properties that an abstract object encodes are precisely those expressed by their defining conditions. . . . We therefore have an answer to Benacerraf's worry that no link between our cognitive faculties and abstract objects accounts for our knowledge of the latter" (547). But if objects that encode differently may have their properties in common this apparently uniform accocunt of reference and knowledge must fail. For differences in the way objects encode are not sufficient place us into knowledge and reference relations — there simply are no relations to particular ones of them.

We address properties in chapter 4. However the obvious response to this worry that some things have all their properties in common is that properties, especially identity properties, emerge in the hierarchy along with things, so that there are properties to distinguish all the things. Insofar as this is so, the result is not Zalta's theory. But, in the spirit of his view, there are objects with property constituents. Thus it is natural to adopt a picture something like the following (I adapt from Jubien, "On Properties and Property Theory"). Begin with a model of ZFC and suppose $\mathcal{O}$ is the set of "ordinals" of the model. Given sets t of proper things, p of properties and k of kinds, represent a property as the set of things to which it applies.

$$T = \bigcup_{\alpha \in \mathcal{O}} T_\alpha \qquad\qquad P = \bigcup_{\alpha \in \mathcal{O}} P_\alpha$$

$$T_\lambda = \bigcup_{\alpha < \lambda} T_\alpha \qquad\qquad P_\lambda = \bigcup_{\alpha < \lambda} P_\alpha$$

$$T_{\alpha+1} = T_\alpha \cup T'_{\alpha+1} \qquad\qquad P_{\alpha+1} = P_\alpha \cup \wp T_{\alpha+1}$$

$$T_0 = t \qquad\qquad P_0 = p \cup \wp T_0$$

In this case, the members of $T'_{\alpha+1}$ are sets of triples $\langle a, \text{R}, \text{K} \rangle$ where $a \subseteq (T_\alpha \cup P_\alpha)$, R is a relation on a and K is a kind. Then things are sets of triples as before, except that their constituents are drawn from both things and properties at levels below. At each stage, properties correspond to sets of things. Let π be a function from members of p to $\wp T$; then P $\in p$ applies to an x just in case $x \in \pi(\text{P})$. So the image corresponding to any property is an associated set where the property applies to things that are its members. Observe that properties in p might apply to things at any stage of the hierarchy. Thus objects with properties as constituents appear along with ordinary things. And properties of these objects, including identity properties, appear along with them as the hierarchy grows.

2.5 Conclusion

This chapter began with consideration of the statue and the clay. That led to an account on which the world is populated by proper things. These have modal properties of the sort BEING SUCH THAT NECESSARILY ANYTHING THAT IS THE SAME K AS IT IS P. Ordinary physical things require also kind properties but as constituents. Once we have them, however, ordinary physical things are a lever into the rest of the platonic universe: Once we admit that there are rocks and trees, dogs and people, by the same account there are events, types, fictional characters and sets. On the present account, then, there are things both abstract and concrete. The resultant view is sketchy in various respects. However, we have enough to suggest that some ordinary things are abstract; so SP according to which there are abstract objects. This does not take us all the way to grounded platonism however. GP requires both that there are some abstract objects, and SN that all objects are grounded in the concrete. There are abstract objects, and they have a ground in ordinary things and kinds. This counts as a concrete ground only insofar as the kinds themselves are so grounded. So a full vindication of GP waits for chapter 4 and our discussion of properties.

Further, our account is at least suggestive of responses to concerns about causation and knowledge for abstracta. Insofar as we grasp features of some proper or ordinary things together with the nature of functions from them to constituted objects, we are in a position to know features of the objects so constituted. We are in causal contact with proper things and so presumably may have knowledge of them. Given that a (proper) person has a particular mother it follows that the (ordinary) person has that mother essentially. It is properties of snow that result in properties of an avalanche; it is properties of floats that result in properties of a parade. And similarly for members of the abstract universe; features of constituting objects, with functions to objects constituted may be sufficient for knowledge. Short of a full-blown analysis of causation, I do not say abstract objects enter into causal relations. However, the functional relation to objects constituted suggests at least the potential *causal relevance* of abstract objects. It is because the character (type) has a certain shape that I am having trouble making out this article. It matters that the set of people in the room is a subset of the set of all people who are my friends. And it matters for seating at a party, say, which sequences have *a* and *b* adjacent but no less than two positions from *c*.

This approach to ordinary things does not exist in a vacuum (though, I regret to say, it was developed almost as though it were). So for example, K. Koslicki, *The Structure of Objects* maintains a view with strong affinities to the one developed here (see also, Fine, "Things and Their Parts," and "Towards a Theory of Part" with

Johnston, "Parts and Principles," and "Hylomorphism"). Restricting her focus to ordinary physical objects, Koslicki offers a theory of parts and wholes in contrast to standard classical extensional mereology. On this standard account, composition is unrestricted so that whenever there are some things, there is a (unique) fusion of them. In contrast Koslicki offers,

RCP *Restricted Composition*: Some objects $m_1, \ldots, m_n$ compose an object O of kind K just in case $m_1, \ldots, m_n$ satisfy the constraints dictated by some formal components $f_1, \ldots, f_n$ associated with objects of kind K. (173)

Formal components specify a form satisfied by the $m_1, \ldots, m_n$. The formal components are like a recipe, and material components ingredients; the ingredients compose an object when they are mixed together according to the recipe. It is not entirely clear that the result in fact restricts composition relative to standard mereology. So, for example, if *arbitrary sum* is recipe which applies to $m_1, \ldots, m_n$ just in case those objects exist, then by RCP there are all the sums of classical extensional mereology — and objects corresponding to other other formal principles as well.[37] Koslicki maintains that the question of what kinds there are is not to be settled by a theory of parts and wholes, but by ontology more generally (171). And she supposes that a commonsense and scientifically acceptable ontology together with her theory rules out the arbitrary combinations of classical extensional mereology.

A distinctive feature of Koslicki's view is that when an object is composed by some $m_1 \ldots m_n$ subject to formal constraints $f_1 \ldots f_n$ the formal components are themselves parts of the object.

NAT *Neo-Aristotelian Thesis*: The material and formal components of a mereologically complex object are *proper parts* of the whole they compose. (181)

So objects have formal along with material parts. Koslicki advances a "master argument" for this thesis: The statue and clay have the same spatiotemporal location and all the same categorical properties; it is thus natural to think the statue has the clay as part. But given modal differences, the statue is not identical to the clay. So the statue has the clay as proper part. But Koslicki accepts a "weak supplementation principle" (WSP) according to which if x has y as a proper part, then x has also some z disjoint from y as proper part. It follows that the statue has a proper part disjoint from the clay. But (at some level of decomposition — at the level of atoms perhaps) the statue and clay have all the same material parts. So the statue has a nonmaterial

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[37]Merricks makes this point in "Review of *Structure of Objects*" (301). In "Towards a Theory of Part" Fine treats such sums as a natural consequence.

part that the clay does not. According to Koslicki this part is the statue's structure. And if the statue is so composed, similarly for other material objects (176-181). On Koslicki's view, "it is precisely when a plurality of objects satisfies the structural requirements imposed on them by a particular kind that a unified thing, a member of the kind in question, results" (237). Apparently, then, an object is composed just in case some $m_1 \ldots m_n$ have an appropriate structure; but is composed only together with that structure as a part. So if some parts are appropriately arranged, there are the statue and the clay; but the statue has its structure as a part, and the clay its structure as a part. These structural parts serve to distinguish the statue from the clay.

Given this account of parts and wholes, Koslicki describes constitution as a species of composition.

MAC *Mereological Analysis of Constitution*: Some objects, $m_1, \ldots, m_n$ *constitute* an object, O just in case $m_1, \ldots, m_n$ are O's material components. (185)

A thing is composed by (all of) its parts, but constituted by its material parts. On the account I offer, the parts of an ordinary thing result from its ordinary properties; beyond this, I have not had much to say about parts. I have imagined, however, that the kinds which are among the constituents of an ordinary physical thing do not appear among its parts (as p. 79). Given this, short some details, Koslicki's view may seem isomorphic to my own: I say ordinary physical things are constituted by some things with a kind property (aa, κ) and may have the things aa as parts. She has the things and kind as parts, but just the things as constituents. Insofar as 'part' and 'constituent' are theoretical terms, it may appear the the one does the work of the other on the different theories so that there is nothing to choose between the different views.

But I do not think this is right. In order to see what is different, let us briefly consider a philosopher SuperK who holds a view like Koslicki's but with application to objects "up up and away" in the hierarchy beyond ordinary physical things.[38] So, again, she says things and their formal features are parts (I would say 'constituents'), but just the things are constituents (I might allow 'parts'). But on my view the constituents upon which a kind operates are not (always) *parts* of the resultant object, and the kind need not be a structure of them. Recall that the constituents of the ACTU are not the same as its member unions (p. 102); supposing that the parts of the ACTU are just the member unions, its constituents are not its parts. Similarly, though some tokens are constituents of a type, a complex object with tokens as parts would not be a type. Intuitively, the parts of a type are certain features of it. Further, BEING A

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[38]Fine, "Towards a Theory of Part," and Johnston, "Hylomorphism" may seem to be views of this sort. So for example both deveop extended applications of their theories of parts and wholes to sets.

FONT TYPE does not apply to the constituents as such. Again, the parts of a fictional character do not include works of fiction or the like that are its constituents; and BEING A FICTIONAL CHARACTER does not apply to such constituents. The kind is a function from the constituents to an object to which the kind applies. Koslicki is able to talk about a "hierarchy" whereby one physical thing is a part of another. And we are imagining that SuperK allows similar structures within categories of events or the like. Insofar as the parts of an object are its constituents, our views may seem to mirror one another. However, insofar as constituents and parts come apart, the views are not the same. In fact, I do not offer a theory of parts at all. So our views may be complementary. My aim has been to say how there are objects in different categories of the hierarchy of ordinary things. Koslicki and SuperK tell us about the way things are related to their parts. However, the story in terms of parts is not the same as one that moves beyond parts in order to move up the hierarchy of ordinary things from one category of thing to another.

There is also room to wonder whether SuperK at least has the role of formal elements properly located. In an extended note, Koslicki considers whether her view is susceptible to a neo-Bradleyan regress on which a structured whole with material components Y and Z and formal component F needs some further formal components to "bind" F with Y and Z. But, says Koslicki, this is like asking whether in order to make a quantity of glue bind together two pieces of paper, we need a further glue to bind the original glue with the paper — "nothing of the sort is required, if the first type of glue is of the right kind to react chemically with paper" (198 n40). Formal components are parts *instantiated* by constituents. Grant that the lump and clay are thereby distinguished. Still, consider $\{a, b, c, d\}$ and $\{\{a, b\}, \{c, d\}\}$. Supposing that sets have their members as parts (and that parthood is transitive, so that parthood for sets is like the ancestral of membership), there is a level of decomposition on which each of these have a, b, c, and d as constituents.[39] And SuperK says they have a formal component from BEING PARTS OF A SET. But then it appears that all their parts are the same.[40] So by the master argument: Given that the parts are the same, it is perhaps natural to think one of the sets is a part of the other; but, reasoning as before, the sets are not identical; so the one is a proper part of the other; and in this case the the weak supplementation principle appears to suggest that one has a part the other does not —

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[39] If a has R to b iff there is an arrow from one to another, $\bullet \to \bullet \to \bullet$ then a has the *ancestral* of R to b iff there is a path along the arrows from a to b. So an individual arrow may indicate a *parent* of a and a path along the arrows an *ancestor*; and on the suggested account a thing is a part of a set so long as there is a "path" along membership relations from one to the other.

[40] Johnston, "Hylomorphism" (675) urges an objection along these lines. Adapting an example from Merricks, "Review of *Structure of Objects*" it may be that an ice cube $\{a, b, c, d\}$ composed of ice cubes $\{\{a, b\}, \{c, d\}\}$ is a physical analog that would apply to Koslicki and not only SuperK.

and this part would seem to be some "glue" to connect the set structure with its parts. In this case, the sethood relation applies in too many combinations for instantiation to do the work of identifying its application. Or, rather, we look in the wrong place for "glue." On our account, objects combine with kinds *at the level of origin* to result in ordinary things — and things so produced have their own identity. Once they are produced, ordinary things may be associated with kinds in different ways; for all I have said, ordinary things may yet have kinds as parts (though I have doubted it). However, the identity of a thing is by its resulting from constituents including the kind, not from the parts instantiating a property.

Further, it is not clear whether it is appropriate to accept the weak supplementation principle, which plays so crucial a role in Koslicki's master argument for the Neo-Aristotelian Thesis prior to an account of things that would make it true.[41] Perhaps it is intuitive that if we subtract a part, like the head of a statue, which is strictly smaller than a whole, there will be some remainder which is disjoint from the subtracted part. However, when we turn to parts that are spatiotemporally coextensive as in the case of the statue and clay, intuitions are less clear. On the current theory, the statue and clay have distinct grounds. However, it is not built into the theory that the statue and clay have different parts — though, again, for all I have said, they might have their different kinds as parts. So the weak supplementation principle is not yet vindicated. Thus Johnston, for example, agues from an account of things and parts to results that would make the WSP false (as Hylomorphism, 664f). At least the WSP is not obviously positioned to drive a theory of parts and wholes.

So we are left with reason to doubt whether structures are parts, and whether structures as parts are positioned to do the work they are meant to do. Perhaps, then, a theory which locates kinds as functions from constituents positions them not only to explain the existence of ordinary things, but for some of the work Koslicki expects of kinds as parts.

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[41] This point is forcefully made by Donnelly, "Using Mereological Principles" who critically considers Koslicki's reasons for accepting the weak supplementation principle, developing alternative principles which might satisfy her rationale for the WSP but without the WSP as a consequence.

Chapter 3

Possibility and Possible Worlds

The previous chapter was driven by a conception of modal foundations: There is a problem about *de re* modality. Together with a perspective about modal foundations, this problem leads to an account of things that builds in resources for a response. The task of the present chapter is to extend that conception of modal foundations, and to place it in a broader setting. Insofar as this setting requires elements of the abstract universe, the chapters are complimentary. The problem of modality derives from a picture according to which the requirements for possibility and necessity exceed the resources of actuality. Thus modality appears as a special difficulty or test case for any theory, as grounded platonism, which would ground possibility and necessity not only in actuality, but in that which is actual and concrete. As we shall see, this project continues into the next chapter on properties.

We have already encountered the possible worlds approach to modality. And the standard semantics of modal logic incorporates a set of "worlds" which may or may not have restricted "access" from one to another. Assignments to basic vocabulary are made at these worlds. An expression is *possible* at a world w iff it is true at some world accessible from w, and *necessary* iff it is true at every world accessible from w. An argument is valid iff no such structure includes a world where the premises are true and the conclusion is not.[1] The worlds approach to modality structurally parallels the standard semantics for modal logic — only with some account of worlds that makes them more than mere indices for the arbitrary assignments of logic. As we have seen, Lewis develops an account on which worlds are things like the universe in which we live, with different ones spatiotemporally and causally isolated. According

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[1] A semantics for modal logic is developed in Kripke, "Semantical Analysis of Modal Logic," Kripke, "Semantical Considerations on Modal Logic" and Hintikka, "The Modes of Modality." Priest, *Introduction to Non-Classical Logic* is an accessible introduction. See also appendix B.

to linguistic ersatzism, worlds are sets of sentences.

Possible worlds, in this robust sense, are either in an ontology or not. But on the one hand, it may be implausible to increase an antecedently accepted ontology with the addition of *worlds* as additional entities; and on the other hand, it may be equally implausible to suppose that anything in an antecedently accepted ontology does the work possible worlds are supposed to do. So the worlds theorist faces a dilemma. Lewis's modal realism may seem impaled on the first horn. On Lewis's view, every way a world can be is a way some world is — where the various worlds are (in some robust sense) *worlds*. Why believe in such a plurality of worlds? Because of theoretical fruit. Lewis gives analyses of modality, counterfactuals, propositions, and more via his account, and he regards this fruit as reason to think the account is right. But, despite the fruit, many philosophers have found it difficult to believe all these worlds exist. As Lewis says, his view is often met with an "incredulous stare." One worries that Lewis's reasons are inadequate to justify the addition of these worlds to an antecedently accepted ontology. In contrast, it might be argued that linguistic ersatzism avoids problems associated with the first horn — only to meet its difficulties on the second. On our account, at least, sets and sentences are relatively unproblematic elements of an already accepted ontology; so if the ersatzer postulates no more than the existence of certain sets of sentences, her ontology is relatively unproblematic. On some such basis, the linguistic ersatzer hopes to duplicate or improve on at least some of Lewis's analyses. The ersatzer hopes to provide all this, and ontological economy too. But linguistic ersatzism has its detractors, and important objections have been raised against it; it is not obvious that linguistic ersatzism gets off the second horn.

In this chapter, I consider the prospects for the worlds approach to modality, with particular attention to linguistic ersatzism. As we shall see, the specifically linguistic character of linguistic ersatzism is neither required nor central to our concerns. Certain complex properties, states of affairs or the like might do as well. Linguistic entities are convenient insofar as there are well-known and straightforward mechanisms for manipulating them. A view on which modal clams are quantifications over some actual entities is of the right *sort* to be compatible with grounded platonism. After some preliminary discussion, I turn to concerns about the nature and then the relevance of such worlds. This will lead to a discussion of grounds for modality, which I find not in worlds, linguistic or otherwise, but in the actual structures of non-modal properties.

3.1 Preliminary Skirmishes

Here is a natural first try at a linguistic account of worlds: say a *world story* is a "maximal consistent" set of sentences of some languge $\mathscr{L}_0$ — where (i) $\mathscr{L}_0$ is a natural language, say English, restricted so that every sentence in a world story has a truth value; (ii) a set of sentences is *maximal* iff for each sentence $\mathscr{A}$ in $\mathscr{L}_0$ either $\mathscr{A}$ or its negation is a member of that set; and (iii) a set of sentences is *consistent* iff it is possible for all its members to be jointly true.[2] On this basis, ⌜there is a possible world at which $\mathscr{A}$ is true⌝ is to be analyzed as ⌜there is a world story of which $\mathscr{A}$ is a member⌝, and similarly for other sentences about worlds. On the usual approach, then, ⌜possibly $\mathscr{A}$⌝ is true iff $\mathscr{A}$ is a member of at least one world story, and ⌜necessarily $\mathscr{A}$⌝ is true iff $\mathscr{A}$ is a member of every world story. In this way, if sets and sentences are themselves respectable entities, we might seem to have the advantages of possible worlds without Lewis's (apparent) ontological excess.

In *Counterfactuals* Lewis objects that accounts of this sort do not allow for enough worlds; there simply are not enough sets of sentences in a natural language to represent all the ways the world could be. Suppose there could be a Euclidean space (as studied in high school geometry), with any combination of points filled by matter; then the number of ways the world can be is at least as great as the number of subsets of the set of all the points in the space. There are continuum many points in a Euclidian space; so there are as many ways the world can be as there are subsets of the continuum many points. But the sentences in a natural language are finite strings over a finite alphabet, and the number of worlds is no greater than the sets of such strings. So by straightforward cardinality considerations, it is not the case that there are as many sets of sentences as there are ways the world can be.[3] So the ersatz strategy misrepresents the possibilities.

Say this is right. In *On The Plurality of Worlds*, Lewis grants that linguistic entities for ersatz worlds need not be sentences of a natural language, and goes so far as to allow that worlds may be composed of a "Lagadonian" language on which each object functions as its own name, and each universal as its own predicate;

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[2]Such an account is suggested by Jeffrey, *The Logic of Decision* (196-7), Jackson, "A Causal Theory of Counterfactuals" and developed more fully by Roper, "Towards an Eliminative Reduction of Possible Worlds."

[3]For $\mathbb{N}$ the set of all natural numbers, the cardinality of the points in a Euclidean space, like that of the real numbers, is $\wp(\mathbb{N})$ the set of all the subsets of natural numbers; so there are $\wp\wp(\mathbb{N})$ sets of points in the space and, on our assumptions, as many ways to distribute matter. But there are just $\mathbb{N}$ finite sequences of a finite vocabulary; and no more ersatz worlds than there are sets of sentences; so the number of worlds is at most $\wp(\mathbb{N})$. So by Cantor's Theorem, there are not as many ersatz worlds as ways the world can be. These basic cardinality claims arise in, say, the first couple of chapters of Boolos, et. al., *Computability and Logic*. Cantor's Theorem is discussed below.

sentences are set-theoretic constructions out of this "vocabulary" along with logical connectives and quantifiers (145ff; compare Swift, *Gulliver's Travels* III.V). On this Lagadonian scheme, there are words and sentences corresponding to nothing ever spoken or contemplated by a human speaker (for example, a sentence about some object in a distant galaxy). Further, we may allow that Lagadonian vocabulary arises in "ranks" corresponding to ranks of things. Then for each α there is an $\mathcal{L}^\alpha$ that takes its vocabulary from rank α, where a sentence is a member of the full Lagadonian $\mathcal{L}$ just in case it is a member of some $\mathcal{L}^\alpha$. Supposing all this, the ersatzer is left with a language of exceptional power. However, Lewis goes on to object that, even so, the ersatzer's language does not have sufficient power to represent properly the facts of modality.

Another objection Lewis raises in *Counterfactuals* is that, insofar as world stories are used in the analysis of possibility, the analysis is circular. A world story is a maximal *consistent* set of sentences; and we have explicated consistency in terms of possibility. On this basis, an account of possibility via world stories is indeed circular. (On Lewis's own view, modal claims quantify over the worlds there *are*, so there is no covert appeal to modality in the specification of the worlds.) One response to this difficulty is to explicate consistency in some way that does not itself involve modality. So, on an approach we have already seen, one might appeal to some *axioms* together with a (semantic or syntactic) account of entailment. Presumably sentences like $\exists x(x$ is a sheep $\wedge\ x$ is a pig) should not appear in world stories. However, if each story includes axioms like $\forall x(x$ is a sheep $\rightarrow \neg x$ is a pig) then it might turn out that consistent world stories are just those sets of sentences that are possibly true without a direct appeal to possibility in the account of the stories. Lewis sees specification of the axioms as the key to this approach; and, as one might expect, he is not satisfied (see p. 39).[4]

In *On The Plurality of Worlds*, Lewis describes an additional problem about local and global descriptions with implications for both the axioms and language.

> We describe the spatiotemporal arrangement and fundamental properties
> of point particles in infinite detail — and lo, we have implied that there
> is a talking donkey, or we have fallen into inconsistency if we also
> say explicitly that there is not. The implication or inconsistency is not
> narrowly logical, thanks to a difference in vocabulary. We need to cover

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[4]For a proposal along these lines, see Roper (52). In *Counterfactuals* (85) Lewis objects that no specification of axioms and syntactic criteria will get the truths of arithmetic right. Roper suggests specification of axioms for arithmetic by means of a "model specification." In *On The Plurality of Worlds* (153) Lewis grants this response.

> it somehow.... We need *connecting axioms*: conditionals to the effect
> that *if* — here follows a very long, perhaps infinitary, description of the
> arrangement and properties of the point particles — *then* there is a talking
> donkey. (155)

For consistency, connecting axioms are required. But, further, these are very long and
perhaps infinitary. In *The Many Faces of Realism*, Putnam emphasizes this second
point. He observes that there is no simple account of color. So, "a red star and a red
apple and a reddish glass of colored water are red for quite different physical reasons.
In fact, there may well be an infinite number of different physical conditions which
could result in the disposition to reflect (or emit) red light and absorb light of other
wavelengths" (5). And similarly for solubility, "there is no reason to think that all the
various abnormal conditions (including bizarre quantum mechanical states, bizarre
local fluctuations in the space-time, etc.) under which sugar would not dissolve if
placed in water could be summed up in a closed [finitary] formula in the language
of fundamental physics" (11).[5] So there are reasons to allow that the Lagadonian
language requires not only the extended vocabulary, but also a grammar extended so
that the sets which are Lagadonian sentences include expressions of infinite length.

Out of these preliminary skirmishes emerges an account of linguistic ersatzism
that can serve as a subject for the further objections to be considered. A world story
is a maximal consistent set of sentences of some language $\mathscr{L}$. $\mathscr{L}$ includes the full
resources of the Lagadonian language. Thus there are names and predicates for every
object and universal. And the grammar accommodates expressions of infinite length.
Thus, given all the ordinary operators and rules, if Π is a set of formulas, so are $\bigwedge\Pi$
and $\bigvee\Pi$, and if Ξ is a set of variables and $\mathscr{P}$ a formula, so are $\forall\Xi\mathscr{P}$ and $\exists\Xi\mathscr{P}$.
Intuitively, $\bigwedge\Pi$ is true iff each $\mathscr{P} \in \Pi$ is true, $\bigvee\Pi$ is true iff some $\mathscr{P} \in \Pi$ is true;
$\forall\Xi\mathscr{P}$ is true just in case $\mathscr{P}$ is true for any assignment of objects to the variables
in Ξ, and $\exists\Xi\mathscr{P}$ is true so long as $\mathscr{P}$ is true for some assignment to the variables
in Ξ. (For additional detail see appendix A; and for a reasonable survey Nadel,
"$\mathscr{L}_{\omega_1\omega}$ and Admissible Fragments," with Dickmann, "Larger Infinitary Languages.")[6]
Consistency is specified by axioms and entailment. As we shall see, this ersatz view

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[5]In this first lecture of *The Many Faces of Realism*, Putnam argues against what he calls 'metaphysical realism'. This overall argument is almost certainly unsound. From the Quinean start, a world-making language is extensional. But this leaves open that it is extensional and infinitary. So a problem about the adequacy of a finite extensional language is not a problem about extensionality as such. For discussion, see chapter 6 of Roy, *About What There Is*.

[6]In this context, it is worth observing that the specific example from footnote 4 does not go through. Let $\Pi = \{x = \bar{n}|n \in \omega\}$, where $\bar{n}$ is $S \ldots S\emptyset$ with n instances of S; then the class of models isomorphic to the standard model of arithmetic is characterised by the Peano axioms together with $\forall x\bigvee\Pi$ — that is, $\forall x[x = \emptyset \lor x = S\emptyset \lor \ldots]$. And given a variable-free term $\bar{d}$ for each d in the domain D of some

is inadequate. However the objections and replies about worlds, so thrashed over by philosophers, push us in the direction of an adequate account of modality.

Before launching into this material, a comment on the starting point: From discussion in the last chapter (and the next) I adopt the stance of a realist about abstract sets and properties. But this realism is constrained in two significant ways, each suggested already in chapter 2. First where a proper class is a collection not identical to any set, I do not presume that there exist (that it is appropriate to quantify over) proper classes. And second, I do not assume that there are essences or haecceities for all the individuals that could but do not exist. Appeal to proper classes or haecceities might result in simplification of the ersatz theory. However, even though there are essences for ordinary things, and we have found sets among the ordinary things, the ordinary things and their essences have their ground in actuality, and the sets do not extend so far as proper classes. So we work within the bounds of what we have.

3.2 The Metaphysics of Worlds

An ersatz world substitutes for a universe, with all its myriads of detail. This puts burdens on the expressive power of the language for the sets of sentences — for the language should be sufficient to describe anything about a world. Thus we have allowed that the world-making language $\mathcal{L}$ is Lagadonian. And similarly, philosophers have been led to say ersatz worlds are maximal — that for any sentence $\mathcal{A}$, either $\mathcal{A}$ or not-$\mathcal{A}$ is a member. I raise concerns related to each point.

3.2.1 Maximality

World stories are required to be maximal sets of sentences. And a requirement of this sort is typically imposed on ersatz "worlds." The idea is to let stories represent everything about ways the world could be (within the expressive limits of the language). There are at least two reasons for requiring this sort of completeness. First, possible worlds are supposed to determine all the modal truths. Without maximality, distinct possibilities might be conflated. Second, a world contains a great many interconnected situations. For a situation (say, a pig's flying) to be possible, it is not enough that a description of it, taken alone, is consistent; it must be that it, together with other interconnected situtations in its world (the density of the pig, the density of air, the

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structure, let Γ be the set of atomics and negated atomics true on the structure, and $\Delta = \{x = \overline{d} \mid d \in D\}$; then the structure is characterized up to isomorphism by $\bigwedge \Gamma \wedge \forall x \bigvee \Delta$. All the same, categoricity results are complex, and the general point of the example remains.

working of gravity, and so forth), is consistent. So a completeness requirement helps insure that consistency is sufficient for possibility.

Unfortunately, there are no maximal sets of sentences in $\mathcal{L}$. Perhaps this is immediate from our metaphysics with things and Lagadonian vocabulary in ranks. But the details of the metaphysics are not required.[7] Suppose that for any proposition $\mathcal{P}$, some sentence expresses $\mathcal{P}$ and some sentence expresses not-$\mathcal{P}$. This much seems requisite from expressive requirements on the language. (Depending on the metaphysics of propositions and the Lagadonian language, perhaps sentences just *are* propositions.) Then the supposition that worlds are maximal is incoherent.

Set a is a *subset* of set b iff every member of a is a member of b. Set a has at least as many members as set b iff there is a one-to-one map from (some of) the members of a onto all the members of b. The *powerset* of a, $\mathcal{P}(a)$ is the set of all the subsets of a. Suppose there is a maximal world w and consider the set $\mathcal{P}(w)$ which has as members all the subsets of w. By Cantor's theorem, there are not as many sentences in w as there are sets of sentences in $\mathcal{P}(w)$.[8] By expressive power, w includes sentences about each member of $\mathcal{P}(w)$ — given that w is maximal, with our assumption that there are sentences to express any proposition and its negation, for any $s \in \mathcal{P}(w)$, among the members of w is one or the other of,

A1 Some member of s is true.

A2 No member of s is true.

So w includes at least one sentence for each member of $\mathcal{P}(w)$ — that is, there is a function from some of the sentences in w onto all the members of $\mathcal{P}(w)$. So w has

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[7]The point is from Jubien, "Problems With Possible Worlds" (307), Grim, *The Incomplete Universe* (91-124), and others. Jubien, *Possibility* (78-82) develops a version of the argument not tied to the specifically set theoretic nature of world stories. He argues that no "world proposition" (or related entity) is both consistent and maximal in the sense that it *entails* every proposition or its negation.

[8]Suppose w has at least as many members as $\mathcal{P}(w)$; then there is a one-to-one map h from members of w onto all the members of $\mathcal{P}(w)$. Since h is a function from sentences to sets of sentences, we may ask if a given sentence $\mathcal{A}$ is itself a member of $h(\mathcal{A})$. Thus, with integers, if some function f has $f(2) = \{2, 4, 6\}$ and $f(3) = \{19, 127\}$, then 2 is a member of $f(2)$ but 3 is not a member of $f(3)$. Consider $c = \{\mathcal{A} \in w \mid \mathcal{A} \notin h(\mathcal{A})\}$, the set of all elements $\mathcal{A}$ in w such that $\mathcal{A}$ is not a member of $h(\mathcal{A})$; c is formed by collecting every sentence $\mathcal{A}$ in world w which is not a member of the subset to which it is mapped by h. Any sentence in c is collected from w; so c is a subset of w and thus a member of $\mathcal{P}(w)$. But c is designed so that it differs from every $h(\mathcal{A})$ in membership of $\mathcal{A}$: Consider an arbitrary $h(\mathcal{A}) = a$; if $\mathcal{A}$ is a member of a, then by construction, $\mathcal{A}$ is not included in c, so $a \neq c$; if $\mathcal{A}$ is not a member of a, then by construction, $\mathcal{A}$ is included in c and again $a \neq c$; either way $h(\mathcal{A}) \neq c$. So there is no $\mathcal{A}$ such that $h(\mathcal{A}) = c$, and h does not map onto all the members of $\mathcal{P}(w)$. This contradicts the specification of h, itself a consequence of the original assumption about w; reject the assumption: w does not have at least as many members as $\mathcal{P}(w)$.

at least as many members as $\wp(w)$. But this contradicts Cantor's theorem; reject the assumption: there is no maximal w. So, given a language with adequate expressive power, the very attempt to say everything about a world is self-defeating.

Given that there are no maximal sets of sentences in $\mathscr{L}$ and that appeal to proper classes (and the like) is ruled out, a natural response is to obtain the function of worlds but without maximality (Humberstone, "From Worlds to Possibilities," Roy, "In Defense of Linguistic Ersatzism"). The idea is to obtain the effect of worlds by means of *partial stories*. On a partial story view, there are some non-maximal sets of sentences, the elements of which can be jointly true; modal language is to be construed as quantification over these entities. Naturally, the argument against maximal sets is irrelevant to sets which are not maximal. It remains however, that a partial story account needs some response to the conflation and consistency concerns that led to maximality in the first place. For this, observe that there is available a sort of surrogate for maximality: even though there is no set containing all the sentences in $\mathscr{L}$, for any sentence in $\mathscr{L}$, there are sets of which it is a member. Given an appropriate modification of the usual semantics, this is sufficient to enable a quantificational account of modal truth.

As a first step toward a partial story account, the notion of a *metaphysically consistent story* is introduced. Our Lagadonian language $\mathscr{L}$ is as before. Let $\mathscr{L}_\kappa$ be $\mathscr{L}^\kappa_{\kappa\kappa}$ — with vocabulary from rank κ, and the cardinalities of any sets Π of sentences and Ξ of variables less than κ. Although there is no set of all the sentences in $\mathscr{L}$, there are sets of the sentences of any $\mathscr{L}_\kappa$. A κ-maximal set — with $\mathcal{A}$ or $\neg\mathcal{A}$ for each $\mathcal{A}$ in $\mathscr{L}_\kappa$, is a *story*. Certain sentences of $\mathscr{L}$ are axioms; it may be that there are axioms from every rank. Entailment is explicated semantically in terms of interpretations, rather than syntactically in terms of derivations. Although derivation systems for infinitary languages may be sound, and complete on certain fragments of the infinitary languages, it is not generally the case that derivation systems for an infinitary language are complete so that if $\Gamma \vDash \mathcal{P}$ then $\Gamma \vdash \mathcal{P}$. Derivations which may be inadequate, are therefore sidestepped by direct appeal to semantic validity.

Consider then the infinitary free logic described in appendix A. It is not important to dwell on the details, though you may decide to check out the appendix. Allowing that the vocabulary of $\mathscr{L}$ may be too large to permit a model, say an α-interpretation is an interpretation of $\mathscr{L}_\alpha$. Then, reverting briefly to the language of plural quantification, sentences are α-*consistent* just in case there is an α-interpretation on which none are false (and so all the ones in $\mathscr{L}_\alpha$ are true). There is a natural plural version of *union* so that, zz are the union of xx and yy just in case z is included in zz iff it is included in xx or yy — $zz = xx \cup yy \leftrightarrow \forall z(z \preceq zz \leftrightarrow z \preceq xx \vee z \preceq yy)$. Say aa are the axioms. Then sentences are metaphysically consistent just in case they together with

the axioms are always α-consistent.

MC Sentences xx are *metaphysically consistent* iff $xx \cup aa$ are α-consistent for every α.

Equivalently, premises α-*entail* $\mathcal{P}$ iff $\mathcal{P}$ is true on every α-interpretation on which none of the premises is false (so $\mathcal{P}$ is true on α-interpretations that make each of the premises in $\mathcal{L}_\alpha$ true). Sentences xx *metaphysically entail* $\mathcal{P}$ just in case $xx \cup aa$ α-entail $\mathcal{P}$ for some α. And sentences xx are metaphysically consistent iff there is no $\mathcal{A}$ such that xx metaphysically entail both $\mathcal{A}$ and $\neg\mathcal{A}$. So for some κ, a metaphysically consistent story is a κ-maximal set such that its members are metaphysically consistent.

Say a story *s extends t* $(s \geq t)$ just in case $t \subseteq s$. Then a metaphysically consistent μ-maximal s has metaphysically consistent extensions at any $\nu > \mu$ [TR4.2].[9] This connects metaphysical consistency to the way consistency was supposed to work on the world story account. There are no world-stories. Still metaphysically consistent stories function as world stories were supposed to work. A world story is supposed to be maximal with $\mathcal{P}$ or $\neg\mathcal{P}$ for each $\mathcal{P}$ in $\mathcal{L}$, and metaphysically consistent — so that the axioms and its members are α-consistent for every α. First, suppose there "would" have been a world story according to which pigs fly. Insofar as it is maximal, it is μ-maximal for every μ, and insofar as it is metaphysically consistent, all the members from $\mathcal{L}_\mu$ together with the axioms are α-consistent for every α. So, where 'pigs fly' appears at some rank μ, the μ-maximal set of sentences from the world story, together with all the axioms, is α-consistent for every α — where this is just to say there would be a metaphysically consistent story according to which pigs fly. Second, suppose there is a metaphysically consistent story according to which pigs fly. Then, given the result about extensions, there is a "chain" of stories, one at each rank, each extending ones before. Then the "union" of stories in the chain would be a world story according to which pigs fly: maximal since for any $\mathcal{P}$ in $\mathcal{L}$, some story in the chain includes $\mathcal{P}$ or $\neg\mathcal{P}$; metaphysically consisent since any sentences from $\mathcal{L}_\kappa$ appear already in a member of the chain such that its members together with the axioms are α-consistent for every α [see TR4.3]. So there is a metaphysically consistent story according to which pigs fly iff there "would" have been a world story according to which pigs fly. And the "jointly possible" claim retains whatever plausibility it was supposed to have on the world story account.

This section is technically messed up

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[9]References in square brackets are to theorem numbers from Roy, "Technical Results for *Properties, Possibilities and Ordinary Things*."

It may be objected that a world story is supposed to be consistent in the sense that there is a model for all its members, and that our reasoning raises a "compactness' concern. Thus it is worth observing that, corresponding to the point about completeness, infinitary logics are not generally compact in the sense that whenever a set Δ of cardinality κ is such that all its subsets of cardinality $< \kappa$ are consistent, so is Δ. For a simple example, let Π be $\{a = c_i \mid i \in \omega\}$ so that $\Pi = \{a = c_0, a = c_1 \dots\}$. And consider a sequence of sets with members,

$$
\begin{array}{ll}
p_0 & \bigvee \Pi \\
p_1 & \neg(a = c_0) \\
p_2 & \neg(a = c_0 \wedge \neg(a = c_1) \\
p_3 & \neg(a = c_0 \wedge \neg(a = c_1) \wedge \neg(a = c_2) \\
\vdots &
\end{array}
$$

Set $s_0 = \{p_0\}$, $s_1 = \{p_0, p_1\}$, $s_2 = \{p_0, p_1, p_2\}$ and so forth. The sets are individually consistent. From p_0, a is identical to one of the c_i. And for any s_n, there is an interpretation on which a is identical to, say, c_n but not to any of c_i for $i < n$. But no interpretation satisfies the union of them all — for on any interpretation, for some i, $a = c_i$ from p_0, and $a \neq c_i$ from, say, p_{i+1}.

We encounter a related situation if the axioms include all the axioms of set theory along with ⌜there are at least κ abstract entities⌝ for each cardinal number κ. Then 'there is a set of all the abstract entities' is consistent with any set of axioms, though they are not jointly possible together. First, suppose the axioms are the axioms of set theory and $\mathcal{A}_\kappa$ for each κ, where $\Xi = \{v_\gamma \mid \gamma < \kappa\}$, $\Sigma = \{abstract(v_\gamma) \mid \gamma < \kappa\}$, $\Pi = \{v_\gamma \neq v_\delta \mid \gamma, \delta < \kappa \text{ and } \gamma \neq \delta\}$, and $\mathcal{A}_\kappa = \exists \Xi (\bigwedge \Sigma \wedge \bigwedge \Pi)$. Then 'there is a set of all the abstract entities' is consistent with any set of axioms, but not consistent with all the axioms taken together. For the set of all the abstract entities must have some cardinality μ, but from an $\mathcal{A}_\kappa$ with $\kappa > \mu$, have a cardinality κ greater than that. In this case, all the axioms have a simple non-logical vocabulary; since there is no model on which each of them together with 'there is a set of all the abstract entities' is true, there is no metaphysically consistent story at which 'there is a set of all the abstract entities' is true. So far, then, so good. Suppose, however, that there are some names of the abstract objects and axioms are of the sort $\mathcal{B}_\kappa$, where $\Sigma' = \{abstract(a_\gamma) \mid \gamma < \kappa\}$, $\Pi' = \{a_\gamma \neq a_\delta \mid \gamma, \delta < \kappa \text{ and } \gamma \neq \delta\}$, and $\mathcal{B}_\kappa = \bigwedge \Sigma' \wedge \bigwedge \Pi'$. If each $\mathcal{B}_\kappa$ appears at a different rank, then there is no $\mathcal{L}^\alpha$ in which they all appear. So far then, a story according to which there is a set of all the abstract entities remains α-consistent for every α, and so metaphysically consistent. And similarly for a world story including the axioms all at once.[10] This seems bad. At this stage, perhaps it is enough to

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[10]Alternatively, one might suggest that if there "were" world stories there would be some mechanism

observe that there *are* ways the axioms could be such that this problem does not arise. So, for example, axioms could be as before, or 'there is no set of all the abstract entities' might be axiomatic, and if axiomatic, would block the problematic sentence. In general, many statements (as that something is a sheep and a pig) may fail to be jointly possible — the point of axioms is to expose such failure. And it is only together with entailment that axioms translate sentences which are not jointly possible into inconsistency. Given the formal limits of entailment, if axioms are inadequate to expose a failure of joint possibility, then axioms are inadequate. This is one reason why we have given such broad scope to the language in which axioms appear. At any rate, it remains that there is a metaphysically consistent story according to which there is a set of the abstract entities iff there is a world story according to which there is a set of them all. So, again, no special worry about the "jointly possible" claim attaches to metaphysically consistent stories.

What about conflation? For world stories, maximality was to prevent conflation of possibilities that $\mathcal{L}$ has the power to distinguish. Although no partial story contains all the sentences in $\mathcal{L}$, it remains that whatever is metaphysically consistent and can be described in $\mathcal{L}$ is described in some metaphysically consistent story. Thus possibility is straightforward: *possibly $\mathcal{A}$* iff $\mathcal{A}$ at some metaphysically consistent story. Necessity is less clear. Stories may "leave out" all sorts of detail, including necessary truths, so that a necessary $\mathcal{A}$ does not appear in in each story. Given the point about extensions from above, for any metaphysically consistent x and sentence $\mathcal{A}$, metaphysically consistent extensions of x include either $\mathcal{A}$ or $\neg \mathcal{A}$. We build on this "maximality" of the story *structure*. Given the way stories are extended, our idea is to let sentences from the full vocabulary of $\mathcal{L}$, and in particular, every necessarily true sentence, be *true* at each metaphysically consistent story, even though not included in each story. Given this, something like the standard approach applies.

Let x, y, and z range over metaphysically consistent stories, v over variables and a over names in $\mathcal{L}$. *Ea* is atomic and abbreviates 'a exists'. For any formula $\mathcal{A}$, $\mathcal{A}^v_a$ is $\mathcal{A}$ with all the free instances of v replaced by a. The operators $\neg$, $\bigwedge$, $\forall$ and $\square$ are primitive. Others are introduced in the usual way. In particular, $\mathcal{A} \rightarrow \mathcal{B}$ abbreviates $\neg \bigwedge \{\mathcal{A}, \neg \mathcal{B}\}$.

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for evaluating consistency of all their members at once, so that a world story according to which there is a set of all the abstract entities is ruled out. Of course, if there were such a mechanism, it should be available for application to partial stories as well — so that there would be no metaphysically consistent story according to which there is a set of all the abstract entities. So, again, the partial story account tracks with the world story one. I thank Phillip Bricker for pressing this "compactness" worry, and posing an example of this sort — though against an earlier version of my view as from, "In Defense of Linguistic Ersatzism."

T1 For any metaphysically consistent x and sentences $\mathcal{A}$ and $\mathcal{B}$,

(B) If $\mathcal{A}$ is atomic, $\mathcal{A}$ is *true*$_1$ at x iff for each $y \geq x$ there is a $z \geq y$ such that $\mathcal{A} \in z$.

(¬) $\neg\mathcal{A}$ is *true*$_1$ at x iff for no $y \geq x$, is $\mathcal{A}$ true$_1$ at y.

(⋀) $\bigwedge\Pi$ is *true*$_1$ at x iff each $\mathcal{A} \in \Pi$ is true$_1$ at x.

(∀) $\forall\Xi\mathcal{A}$ is *true*$_1$ at x iff for any $\Pi = \{Ea \to \mathcal{A}_a^v \mid v \in \Xi\}$, $\bigwedge\Pi$ is true$_1$ at x.

(□) $\Box\mathcal{A}$ is *true*$_1$ at x iff for any y, $\mathcal{A}$ is true$_1$ at y.

We shall not have much cause to appeal infinitary expressions true at worlds — the main role for infinitary expressions is in world-making. Still, the semantics makes room for such expressions. Conditions for other operators result in a natural way.[11] Let a story be *actually* true iff each sentence in it is true of the actual world. $\mathcal{A}$ is *true* iff it is true$_1$ at some actually true story.

For any metaphysically consistent story, there are sentences such that neither they nor their negations are true$_1$ at it. But these "gaps" are merely the gaps naturally associated with story incompleteness. A metephysically consistent story may fail to pronounce on the truth or falsity of some $\mathcal{A}$; but it does not thereby pronounce $\mathcal{A}$ neither true nor false. Other results are standard. A non-modal $\mathcal{A}$ is true at x just in case every $y \geq x$ has a $z \geq y$ such that $\mathcal{A} \in z$; so such a sentence is true at a story so long as, no matter how the story is extended, the sentence always shows up eventually [TR4.5]. No matter how many of them there are, necessary truths are true at extensions of every story; so all the necessary truths obtain at each story. As was supposed to be the case on the world story account, then, a non-modal sentence is necessarily true iff it is true$_1$ at each metaphysically consistent story; it is possibly true iff it is true$_1$ at some metaphysically consistent story. Supposing that any non-modal sentence is true or false in some actually true story any sentence is either true or false [TR4.6]. Say an argument is *logically valid* just in case there is no metaphysically consistent story where the premises are true and conclusion false. Then a standard free quantified S5, F1v described in appendix B is sound [TR4.7].

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[11]Thus $\mathcal{A} \wedge \mathcal{B}$ is *true*$_1$ at x iff $\mathcal{A}$ is true$_1$ at x and $\mathcal{B}$ is true$_1$ at x; $\mathcal{A} \vee \mathcal{B}$ is *true*$_1$ at x iff for each $y \geq x$ there is a $z \geq y$ such that $\mathcal{A}$ is true$_1$ at z or $\mathcal{B}$ is true$_1$ at z; $\mathcal{A} \to \mathcal{B}$ is *true*$_1$ at x iff for each $y \geq x$, $\mathcal{A}$ is not true$_1$ at y or $\mathcal{B}$ is true$_1$ at y; $\exists v \mathcal{A}$ is *true*$_1$ at x iff for each $y \geq x$ there is a $z \geq y$ and a such that Ea and $\mathcal{A}_a^v$ are true$_1$ at z; $\forall v \mathcal{A}$ is *true*$_1$ at x iff for any a and $y \geq x$, Ea is not true$_1$ at y or $\mathcal{A}_a^v$ is true$_1$ at y; and $\Diamond\mathcal{A}$ is *true*$_1$ at x iff for some y, $\mathcal{A}$ is true$_1$ at y. And for the infinitary operators, $\bigvee\Pi\mathcal{A}$ is *true*$_1$ at x iff for each $y \geq x$ there is a $z \geq y$ such that some $\mathcal{A} \in \Pi$ is true$_1$ at z; and $\exists\Xi\mathcal{A}$ is *true*$_1$ at x iff for each $y \geq x$ there is a $z \geq y$ such that for some $\Pi = \{Ea \wedge A_a^v \mid v \in \Xi\}$, $\bigwedge\Pi$ is true$_1$ at z [TR4.4].

As an application of our approach, consider a somewhat different "maximality" objection from Kit Fine ("The Problem of Possibilia," 169-71). Suppose that there can be "telepathic Cartesian egos" and,

(a) There could be at least one Cartesian ego.

(b) If it is possible that there are some egos, it is possible that for any (sub)set of them, an ego is in telepathic communication with just the egos in that set.

(c) If some egos can exist, it is possible that they all exist together.

From (c) there is a story (Descartes's story) according to which all possible egos exist, and from (a) according to Descartes's story there are some egos. With Cantor's Theorem, from (b) one might conclude,

(d) If it is possible that there are some egos, then possibly there are more egos than that.

So there is a story according to which there are more egos than there are according to Descartes's story and, contrary to hypothesis, Descartes's story does not build in all the possible egos. But (d) follows from (b) only with the assumption that the egos that exist according to any given story comprise a set. The immediate consequence of the Cantorian argument is that any set has more subsets than members; if there can be some set of egos, by (b) there can be an ego corresponding to every subset of it; so there can be more egos than are in the set. On our assumptions, however, there need not be a set of all the egos. Suppose (a), (b) and (c) are true at Descartes's story d and there is a set s of egos according to some story $x \geq d$; then there is a $y \geq x$ according to which there are all the egos in s and for any subset of s some ego is in communication with just the egos in that set. Say according to y the egos comprise a set t; then there is a $z \geq y$ according to which there are all the egos in t and for any subset of t some ego is in communication with just the egos in that set. And so forth. So it is true at d that nothing is a set of all the egos, $\neg \exists s \forall x (x$ is an ego $\rightarrow x \in s)$. And this seems right, given the assumptions with which we began. All the possible egos appear eventually in extensions of Descartes's story, so according to Descartes's story the egos from all of the sets exist; but no extension of Descartes's story includes a set of them all. So attempts to show that ersatz entities (stories in this case) are "too small" to include everything are correct, but the force of them is undercut by allowing the ersatz entities to extend, just as a universe of sets (or egos) itself extends under Cantorian pressures.

3.2.2 Expressive Power

Suppose this quantificational approach to modality adequately utilizes the expressive power of $\mathcal{L}$. Still, Lewis (among others) objects that the resources of the *language* are not sufficient to distinguish all the possibilities there are. One form of this objection has to do with distinguishing qualitatively indiscernible individuals. We are to think of worlds where nothing exists except symmetrically arranged qualitatively identical spheres, or worlds of two-way eternal recurrence. The objection is that, in some sense, the language cannot distinguish distinct objects in worlds of this sort.

> According to an ersatz world that represents such repetition in time or space, there are many indiscernible individuals. But we do not have correspondingly many indiscernible ersatz possible individuals, all actualised according to this ersatz world. One must do for all. What the ersatz world says, or implies, is that one ersatz individual is actualised many times over. So where we ought to have many indiscernible possibilities for an individual, we have only one. (Lewis, *On The Plurality of Worlds*, 157-8)

Lewis seems to think of "ersatz possible individuals" as certain sets of sentences that may or may not be actualized according to (world-)stories. But I see no reason to think the ersatzer need be directly concerned with "individuals" of this sort (at least with respect to modality, T1 makes no appeal to such objects). The issue, I take it, is what individuals can be represented by a story as existing. And, in particular, the issue is whether stories can represent there being distinct but qualitatively indiscernible individuals. It strikes me that one ought to be initially suspicious of a claim that stories cannot represent such individuals. To take a familiar example, suppose I am confronted with two intrinsically indiscernible globes a and b; seemingly, I can tell a story according to which nothing but a and b exist, a story according to which nothing but a exists, and so forth. Such examples suggest the obvious initial response that qualitatively indiscernible objects can be distinguished by giving them distinct names.

Lewis anticipates something like this move and responds as follows,

> You might say: 'if multiplicity is wanted, no sooner said than done — let's make many ordered pairs, pairing the one linguistic ersatz individual with each of the infinitely many integers.' But multiplicity was not all I wanted. This is an irrelevant multiplicity. . . . The many representations do not represent the many possibilities unambiguously, one to one. Rather, each of the many new representations is ambiguous over all the many possibilities, just as the one original representation was. Nothing has been gained. (158)

Once again, I take it that the appropriate concern is not with "ersatz individuals" construed as certain sets of sentences, but with the individuals that can be represented as existing by a story. Suppose a story represents there being two-way eternal recurrence, pairing each age with one of the infinitely many integers. Is *this* multiplicity irrelevant? The numeric naming scheme would be adequate to allow a story to say that there are many qualitatively indiscernible individuals, one coming in an age after the other. A story could contain sentences like, 'There are ω ages', 'Napoleon$_n$ lives in age n', 'Age n is prior to age $n + 1$' and so forth. But, further, if according to the story the actual Napoleon lives in some age of the world, why not stipulate that we are talking about *him* and index the various individuals that fill the Napoleonic role to the actual one? (Let the actual Napoleon be Napoleon$_0$; the one that lives in the age after him Napoleon$_1$; and so forth.) As with globes, our ability to distinguish indiscernible objects seems as good as our ability to name them. But maybe no actual object exists in any of the recurring ages.

Perhaps, then, the problem is not differentiating the various "Napoleons" in worlds of two-way eternal recurrence, but differentiating non-actual indiscernible objects. In this case, there is less plausibility to the claim that we can stipulate that we are talking about a particular individual. In a related discussion, Lewis presses this point.

> If the extra individuals do not exist for us to name (or for us to declare them to be names of themselves) how can we possibly have any names for them? And without names for the extra individuals, how do we distinguish ersatz worlds that do not differ at all in what roles are played — in what sort of individuals there are, with what properties, how related — but differ only in which extra individuals occupy those roles? (158)

On Lewis's own view, once one has described the roles that are filled — what kinds of individuals there are and how they are related — one has said everything there is to say about a world; there is no point to saying by name who is who. The objection is thus supposed to apply against the "haecceisist" who thinks it is important to say who is who. On the one hand, then, if we are content with saying what kinds of individuals there are and how they are related, there is no problem with naming non-actual objects. But on the other hand, if we are not content with just saying what kinds of individuals there are and how they are related, why not let the haecceities themselves function in the Lagadonian language as names for the objects of which they are haecceities? If we do this, the language will be able to distinguish the objects.[12]

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[12]Plantinga, "Actualism and Possible Worlds" (268) suggests a response along these lines, though his appeal is to essences — which are not obviously the same as haecceities.

But we have agreed not to claim that there are haecceities or the like for all the objects that could but do not exist. If this is right, the Lagadonian language will not have the resources to name all the non-actual individuals. Thus, in a much-discussed example, Alan McMichael objects that the ersatzer cannot represent the possibility that JFK have had a second son who becomes a senator, but who could have become an astronaut ("A Problem for Actualism"). It is simple enough to tell one story according to which JFK has a second son who becomes a senator, and another where he has a second son who becomes an astronaut. But just as someone other than John Jr. might have been the first son of JFK, so different individuals might have been JFK's second son. And even a Lagadonian language does not name a second son of JFK. So given the one story according to which JFK has a second son who becomes a senator, the other is not one according to which *he* — the very same individual — becomes an astronaut. So we do not represent the possibility that the individual who becomes a senator could have become an astronaut.

On sort of reply (Roy, "In Defense of Linguistic Ersatzism" and, for an account along the same lines, Sider, "The Ersatz Pluriverse") appeals to arbitrary names as placeholders for non-actual individuals. The Lagadonian language $\mathcal{L}$ has no names for "extra" individuals. So extend $\mathcal{L}$ to a language $\mathcal{L}^*$ with the addition of some extra constants. Perhaps, for arbitrary markers N and n, for any thing t, $\langle N, t \rangle$ names t, and $\langle n, t \rangle$ is a name for an "extra" individual. Then $\mathcal{L}^*$ has at least as many names for extra individuals as there are sets: $\langle n, \phi \rangle$, $\langle n, \{\phi\} \rangle$, and so forth. Tell stories in the extended language. Then it might be that 'a is a second son of JFK who becomes a senator' is true at some x and 'a becomes an astronaut' is true at y. Since there is a story where a becomes an astronaut 'a is a second son of JFK who becomes a senator but could have become an astronaut' is true at x. On this scheme, extra constants work as placeholders for individuals; stories in $\mathcal{L}^*$ use the constants to indicate the places. But the extra constants do not name anything; they rather indicate places something could occupy. So it is generalizations in the original $\mathcal{L}$, indicating just the places, which represent real possibility. Thus generalizing on a, 'something is a second son of JFK who becomes a senator and could have become an astronaut' is true at x; and since there is such a story, 'possibly something is a second son of JFK who becomes a senator and could have become an astronaut' is actually true. Similarly, one might tell stories according to which there are qualitatively identical but distinct Napoleons.

Lewis's point remains: If some story in which a plays a role were actualized, the individual filling a's role would be a particular individual — one to whom the story does not refer. But, plausibly, this is just right: The possibility that actually exists is that the role be filled, not that the particular individual who would exist if the story were actualized (and *who is* this?) fill the role. Perhaps there are possibilities for a

particular child JFK never had by virtue of possibilities for some particular (actual) sperm and egg. The difficulty is for objects completely foreign to actuality. Insofar as one claims that possibly there are some particular but non-actual individuals such that *those individuals* would exist if their possibility were actualized, he gets the facts of modality wrong. The worlds hypothesis is run amok.

Suppose it is possible that JFK have had a second son, and that a is introduced as a placeholder name for him. It is natural to think that this son could not have been a rock. So, with Rx for 'x is a rock' and Hx for 'x is human', we expect $\neg Ra$, or for the same result, Ha and $\forall x (Hx \to \neg Rx)$ in stories. Somehow, then, axioms must constrain sentences containing a. Plausibly this is a general phenomenon; axioms must constrain sentences containing names, including sentences containing "extra" names. One option is that $\neg Ra$ or Ha and the like are themselves axiomatic. For things that actually exist, maybe axiom specification can take advantage of the way things actually are ('For any set x, if $y \in x$, let that $y \in x$ be axiomatic'). But for extra names there is no such luxury. And, plausibly, we require as many extra names as there can be Fine's telepathic egos. So there is no short list. Short of god's brute pointing to indicate sentences to be the axioms, it is hard to see how this would go. Even pointing may be difficult if, given all the egos and the like, there is not even a *set* of sentences at which to point. And we have already expressed doubts about the efficacy of this "brazen" ersatz strategy; an arbitrary class of sentences, stack of books or whatever is not of the right *sort* to be a ground for modality (p. 39).

I take up grounds for modality in the following. However we have already seen reason to accept principles like,

Oe $\quad \forall z \forall_Q [\exists x \exists_K (Czx_K \wedge CQx_K) \to \Box \forall w (IQw \leftrightarrow w \approx z)]$

P2$'$ $\quad \forall_Q [\exists x (CQx_P \wedge I_Fx) \to \Box \forall y (IQy \to I_Fy)]$

If z and Q are constituted by x and K, then necessarily anything with Q has all the same categorical properties as z; and if Q is constituted by (x, P) and x is female, then necessarily if a thing has Q it is female. So the way things actually are constrain the way they are in other worlds. But similarly, if according to some story Cax_P and CQx_P, then necessarily, anything with Q has the same categorical properties as a; and from related principles, necessarily nothing with Q is a rock. So from general principles, the way a extra thing is according to one story constrains the way it is in all. Thus stories are required to be consistent relative to *cross-story* constraints along the lines of those above.

As before, I begin with the entities and then move to truth. Let a *book* be any set of stories, now in the extended language $\mathcal{L}^*$. A book X *extends* book Y ($X \geq Y$)

just in case $Y \subseteq X$. We shall be concerned with *book consistency* — where this incorporates the cross-story constraints. So we require some means for sentences in one story to constrain those in another. In appendix B, I describe a system $F1\upsilon$ for free quantified modal logic. Things work in the usual way: An interpretation is $\langle W, U, D, A, V \rangle$ where W is a set of "worlds," U a universe of objects, D a function from worlds in W to their domains from U, A an access relation, and V an assignment to basic elements of the vocabulary. Then sentences are true and false at worlds so that for $w \in W$, $V[w, \mathcal{P}] = 1$ or $V[w, \mathcal{P}] = 0$. For Γ a set of sentences, $V[w, \Gamma] = 1$ iff for each $\mathcal{A} \in \Gamma$, $V[w, \mathcal{A}] = 1$. Then, where the members of Γ and $\mathcal{A}$ are sentences,

VF1υ $\Gamma \vDash_{F1\upsilon} \mathcal{A}$ iff there is no F1υ interpretation $\langle W, U, D, A, V \rangle$ and $w \in W$ such that $V[w, \Gamma] = 1$ and $V[w, \mathcal{A}] = 0$.

The details need not detain us here (though you can peruse them if you like). What matters is that this notion of validity is capable of generalization (anticipated already in rules for the associated derivation system NF1υ). Introduce *subscripts* and expressions of the sort $\mathcal{A}_s$ and $s.t$. Intuitively s and t indicate worlds, where $\mathcal{A}_s$ just in case $\mathcal{A}$ at world s, and $s.t$ when world s has access to world t. Then the standard notion of validity generalizes to accommodate premises and conclusions from multiple worlds. Given a model $\langle W, U, D, A, V \rangle$ let m be a map from subscripts into W. Then say $\langle W, U, D, A, V \rangle_m$ is $\langle W, U, D, A, V \rangle$ *with* map m. Then, where Γ is a set of expressions of the language with subscripts, $V[m, \Gamma] = 1$ iff for each $\mathcal{A}_s \in \Gamma$, $V[m(s), \mathcal{A}] = 1$, and for each $s.t \in \Gamma$, $\langle m(s), m(t) \rangle \in A$. Then for $\mathcal{A}_s$ a subscripted sentence, and Γ a set of such sentences,

VF1υ* $\Gamma \vDash^*_{F1\upsilon} \mathcal{A}_s$ $(s.t)$ iff there is no F1υ interpretation $\langle W, U, D, A, V \rangle_m$ such that $V[m, \Gamma] = 1$ but $V[m(s), \mathcal{A}] = 0$ $(\langle m(s), m(t) \rangle \notin A)$.

Arguments may have premises and conclusion according to which sentences in different worlds are true. Of course, this notion reduces to the ordinary one when all the subscripts are the same.

This is just right, given our concerns. Consider a very simple example. Working in the associated derivation system, suppose we begin with one story according to which Ea and Ha and another according to which Ra, and have a general axiom according to which $\forall x (Hx \rightarrow \Box \neg Rx)$. Associate sentences in the first story with subscript 1 and in the second with subscript 2. Though in each case, the story plus the axiom is individually consistent, as exhibited by a simple derivation, the combination is not.

$$
\begin{array}{r|l l}
1 & Ea_1 & \text{P} \\
2 & Ha_1 & \text{P} \\
3 & \forall x(Hx \to \Box\neg Rx)_1 & \text{P} \\
4 & Ra_2 & \text{P} \\
5 & (Ha \to \Box\neg Ra)_1 & 3,1\ \forall\text{E} \\
6 & \Box\neg Ra_1 & 5,2\ \to\text{E} \\
7 & 1.2 & \text{AM}\upsilon \\
8 & \neg Ra_2 & 6,7\ \Box\text{E}
\end{array}
$$

So both Ra_2 and $\neg Ra_2$ and the supposition that there are the expressions with which we began is inconsistent. Intuitively, if a is human according to one story, and nothing that is human can be a rock, there cannot be another story according to which Ra.

On this basis, it is easy to state the conditions for book consistency. Where the members of X are sets of sentences, associate each $x \in X$ with a unique subscript, and sentences in each x with the subscript for x; $\bigcup_s X$ is the set of all subscripted sentences from sets in X. Subscripted sentences are α-consistent iff all the ones from $\mathcal{L}^\alpha$ have a model. Axioms are some (maybe modal) sentences. Associate each axiom with every subscript from X, so that all the axioms are associated with any story.

CB A book X is *metaphysically consistent* iff $\bigcup_s X$ together with all the axioms is α-consistent for every α.

The members of a book are μ-maximal stories; so there is some size such that no member of a book is greater than it. A book is metaphysically consistent so long as it remains α-consistent when its stories are augmented by all the axioms in any $\mathcal{L}^\alpha$. From the example above, a book whose stories include sentences from $\{\{Ea, Ha\}, \{Ra\}\}$ is not metaphysically consistent in the presence of the axiom $\forall x(Hx \to \Box\neg Rx)$.

Suppose the axioms consist entirely of general principles and so contain no names. Presumably, there is no consistent book containing one story according to which Lewis is a human and another according to which he is a rock. Still, with no names in the axioms, a book containing only stories according to which he is a rock is no less metaphysically consistent than one containing only stories according to which he is human. This seems bad! But a solution is at hand.

PB A book X is *possible* iff for any Y containing only actually true stories, $X \cup Y$ is metaphysically consistent.

If Lewis is human, no possible book contains a story according to which he his a rock. But if a possible book contains a story according to which an "extra" a is human then it (and each possible extension of it) contains no story according to which a is a rock; and if a possible book contains a story according to which a is a rock then it (and each

possible extension of it) contains no story according to which a is human. So the way a character is "introduced" into a book constrains other stories about it.

It remains to give a truth condition for sentences at stories in possible books. The condition is analogous to what we have seen, with the exception that the relevant ways in which a story in a book can be extended are limited to "bookwise" extensions of it. Let 'X', 'Y' and 'Z' range over possible books, and 'x', 'y' and 'z' over stories in X, Y and Z respectively. For sentences in $\mathcal{L}^*$,

T2 For any X and x,

(B) If $\mathcal{A}$ is atomic, $\mathcal{A}$ is *true$_2$* at x in X iff for each $Y \geq X$ and $y \geq x$ there is a $Z \geq Y$ and $z \geq y$ such that $\mathcal{A} \in z$.

($\neg$) $\neg\mathcal{A}$ is *true$_2$* at x in X iff for no $Y \geq X$ and $y \geq x$, is $\mathcal{A}$ true$_2$ at y in Y.

($\bigwedge$) $\bigwedge\Pi$ is *true$_2$* at x in X iff each $\mathcal{A} \in \Pi$ is true$_2$ at x in X.

($\forall$) $\forall\Xi\mathcal{A}$ is *true$_2$* at x in X iff for for any $\Pi = \{Ea \to \mathcal{A}_a^v \mid v \in \Xi\}$, $\bigwedge\Pi$ is true$_2$ at x in X.

($\Box$) $\Box\mathcal{A}$ is *true$_2$* at x in X iff for each $Y \geq X$ and y, $\mathcal{A}$ is true$_2$ at y in Y.

As suggested above, this works very much as before, except for the restriction to "bookwise" extensions. With the usual definitions, $\Diamond\mathcal{A}$ is *true$_2$* at x in X iff for some $Y \geq X$ and y, $\mathcal{A}$ is true$_2$ at y in Y [TR5.3]. Say $\mathcal{A}$ is *X-true* iff for each $Y \geq X$ there is a $Z \geq Y$ and actually true z such that $\mathcal{A}$ is true$_2$ at z in Z. So a sentence is X-true just in case, no matter how X is extended, the result eventually includes an actually true story according to which it is so. Then a sentence $\mathcal{A}$ from the original language $\mathcal{L}$ without extra constants is *true* iff it is X-true for any X. True sentences are ones from the original language that come true at an actually true story in any possible book. Free quantified S5 remains sound [TR5.6].

With Sx for 'x is a senator' and Ax for 'x is an astronaut' we might have $\Diamond Sa$, $\Diamond Aa$ and $\Diamond\exists x(Sx \wedge \Diamond Ax)$ all X-true for a possible book X. But if a is an extra name, $\Diamond Aa$ and $\Diamond Sa$ are not true simpliciter. It is the the generalization, not the sentences with extra names, that is true. But no possible book contains a story according to which Lewis is a rock. So, presumably, each possible book can be extended to contain stories according to which he is a senator or he is an astronaut. And if this is so, then it is true that $\Diamond$(Lewis is a senator) and $\Diamond$(Lewis is an astronaut).

There are related objections about properties that are indiscernible or could but do not exist. But the response is similar. Adopt a second order language or, equivalently, a multi-sorted language with an instantiation predicate. On the latter option, where the Lagadonian language includes names for properties and an instantiation predicate,

so that the logic for consistency includes quantification over properties, continue with books and stories as before (for such a logic, see appendix C). Thus, for example, say C QYzR when Q is constituted relationally from a binary property Y and object z — as BEING TALLER THAN JIM is constituted by BEING TALLER THAN and Jim. Then perhaps there are constraints on relational properties so that,

Rp $\forall$Q$\forall$Y$\forall z[C\text{QY}z\text{R} \leftrightarrow \Box\forall w(I\text{Q}w \leftrightarrow I\text{Y}wz)]$

If Q is bearing Y to z, then necessarily any w has Q iff it bears Y to z. Given this we could tell stories about a property TK such that necessarily a thing has it iff that thing is taller a, Kennedy's second son. Then, generalizing, possibly there is an x and Y such that x is Kennedy's second son and necessarily a thing has Y iff it is taller than x. Again, possibility and necessity appear in the original $\mathscr{L}$. In this case, the multivlued free modal logic MF2υ from appendix C is sound [TR5.7]. And we are in a position to contemplate the possibility that there exist properties not identical to any that are actual.

Thus this proposal plays out the notion that there is a possibility that roles be filled by "extra" individuals, but not that there are some particular individuals to fill the role. I think the proposal is pleasing insofar as it connects with the "partial" and "relative" whay philosophers introduce extra individuals: We say "let a be a horse. . ." never naming or mentioning what a ate for breakfast or the like; and we say what is and is not a possibility for a given (or relative to) the way *'a'* is first introduced. Perhaps, then, we have a reasonable response to the expressive power objections with which we began.

3.3   The Relevance of Worlds

So far, we have addressed technical problems with technical solutions. If ersatz entities do not properly represent the facts of modality, then ersatzism cannot hope to provide an adequate analysis of modality. Suppose such problems are resolved, and some ersatz entities do adequately represent the facts of modality. Even so, there are reasons to question the adequacy of a worlds approach. Perhaps worlds exist as advertised, but still do not solve the problem. So one may ask how or why worlds are so much as relevant to modal truth.

The difficulty is especially related to one of the strands underlying the problem of modality. We expect that any property has an actual categorical ground (AC). But then the role of other worlds is called into question. The worlds theorist faces a dilemma: Either possible worlds are constrained relative to the actual world or they are not.

Suppose, first, that they are not; then it is hard to see how other worlds can be relevant to modal truths which are themselves constrained relative to actuality. Arguably, unless the way other worlds (whatever they may be) are is somehow connected to the actual world, the other worlds will turn out to be irrelevant to modal truths about things in the actual world. But, second, if other worlds are constrained relative to the actual world, it may look as though the other worlds are superfluous, if the modal facts are themselves prior to the worlds.

Consider Lewis's modal realism. Lewisian worlds exist independently of one another; it is therefore difficult to see what they have to do with what is actually possible. Appropriating a case from Jubien's "Problems With Possible Worlds," the problem for Lewis may be dramatized as follows. Suppose I could have been in another line of work. Then, on Lewis's view, there are some (concrete) worlds where I have a counterpart in another line of work. Now suppose that each of these worlds is such that it is annihilated by god at its counterpart to now. Then there is, in the next instant, no world where I have a counterpart in another line of work. Does this change the fact that I could have been in another line of work? I think not — for nothing relevant has changed. Somehow, the way things actually are is sufficient for the possibility that I could have been in another line of work. Of course there are various responses. So, for example, Lewis might bite the bullet, insisting that *the worlds* are the determiners of modality and that if god does this, he eliminates a possibility. But this seems wrong. There would, at least, be nothing new in actuality to prevent my being in another line of work. Lewis might argue that god *could not* annihilate the worlds (maybe there is no "Leibnizian" god outside of the worlds, or maybe the worlds are indestructible). But this does not seem right either. Notice that the thought experiment may be modified so as to depend only on forces internal to worlds (for example on nuclear explosions and/or the sudden deaths of counterparts); also, Lewisian worlds seem to be the wrong *sort* of objects for it to be the case that god could not annihilate them — if Lewis claims the worlds cannot be annihilated or be different for internal reasons, he gets the facts of modality wrong. At any rate, returning to the question of relevance, Lewis's proposal about worlds seems inconsistent with the nature of modal facts.

The first horn of our dilemma has application to theories, like Lewis's that take possible worlds as primitive for modality. Seemingly, the modal facts must determine which things (if any) are the worlds rather than the other way around. But perhaps it is more common for worlds to be introduced in modal terms. Perhaps stories are consistent in the sense that it is *possible* for their members to be jointly true. We have let stories be consistent relative to axioms. But the same point applies if, as Lewis charges, the axioms are themselves specified in modal terms: the ersatzer "can only

declare: the axioms shall include whichever sentences of such-and-such form are necessarily true. Once he says that, all his analyses from there on are modal" (*On The Plurality of Worlds*, 154). On this basis, there is a sense in which worlds are relevant to modal truth: if there is a world at which $\mathcal{A}$, *of course* it is possible that $\mathcal{A}$. But why introduce the worlds at all? Consider this analogy: You are informed by some philosopher that whenever one statement folows from another there is a special abstract entity, a *foof*, corresponding to that connection; you are told that foofs are "last-sentence-following-from-first-sentence" sequences of sentences; and truth conditions for what follows from what are developed in terms of foofs ($\mathcal{A}$ *follows* from $\mathcal{B}$ iff there is an $\mathcal{A}$-following-from-$\mathcal{B}$ foof). But you are never told more than this about foofs. Now why should not you treat this as (perhaps) an interesting claim about the population of the abstract universe, but as uninteresting relative to your concern about what follows from what? For the FOLLOWS FROM relation is prior to a sequence of sentences' foofhood — and not the other way around. Similarly, might not a philosopher object that the ersatzer has (perhaps) made an interesting claim about the population of the abstract universe, but that his claim is uninteresting with respect to modality — the modal facts being what determines which states of affairs are the possible worlds?

Now the proof theorist may hold that there is nothing false in what the foof theorist says, and insist only that there is more to the story than the foof theorist gives and that this something is essential to the interest of the account. And similarly the situation for worlds might be different if one were to say something more about why certain entities represent the possibilities. In this section, I take on the dilemma at its second horn, allowing that stories are constrained by actuality, but arguing that there is more to be said — that there is an important sense in which worlds are relevant to modality. Before turning to a positive account, however, I take some steps toward focusing the worry with some themes from M. Jubien's recent *Possibility*.

3.3.1 Jubien

This same dynamic on which worlds may be irrelevant on the one hand because disconnected from actuality, and on the other because directly tethered emerges in a critique of worlds from Jubien's book. To set the stage, Jubien emphasizes that the "worlds" of semantics for modal logic are (mere) indicies in a mathematical structure. By means of the structure, there are results for the validity and invalidity of arguments. But the mathematics of the logic is not metaphysics. A theorist who accepts the "central tenet" of worlds theory according to which a thing is possible iff true in some possible world, adopts a position structurally parallel to the standard semantics for

logic. But this is not a mathematical justification for the central tenet. Rather, a thesis about the metaphysics of modality requires metaphysical justification. In particular, the central tenet "cannot rise to the status of an analysis of the notion of possibility until we have been told *what the possible worlds are like and why what goes on in other possible worlds has anything to do with what is true in a given possible world*" (69-70). Both Lewis and the ersatzer make headway toward accounts of what the worlds are like. This leaves the question about what other worlds have to do with what is true in ours. And, though worlds might reflect modal reality, Jubien maintains that they do not ground it.[13]

Against Lewis, Jubien invites us to consider the "pure, untitled ontological picture of detached *realms*" (62). Suppose there are two such realms, or twenty-seven. Say the latter, but all of them include stars. On the face of it, the detached realms are not so much as relevant to the question whether stars are necessary beings. Similarly for a stack of twenty-seven books, each asserting that there are stars. So far, the view lacks "modal oomph" to connect the realms to what can and must be. Lewis argues from theoretical fruit to the result that for every way a world could be some detached realm is that way. This fruit requires treating the realms as possibilities. But even the fruitful hypothesis that detached realms are the possibilities does not itself remove the question how or why this is so.

Lewis's counterpart theory may seem to provide the relevant oomph. On this theory, different worlds represent our world as being other than it is. So for example (parts of) worlds represent Hubert Humphrey, the former Vice President and 1968 United States presidential candidate.

> Humphrey may be represented *en absentia* at other worlds, just as he may be in museums in this world. The museum can have a waxwork figure to represent Humphrey, or better yet an animated simulacrum. Another world can do better still: it can have as part a Humphrey of its own, a flesh-and-blood counterpart of our Humphrey, a man very like Humphrey in his origins, in his intrinsic character, or in his historical role. By having such a part, a world represents *de re* concerning Humphrey — that is, the Humphrey of our world, whom we as his worldmates may call simply Humphrey — that he exists and does thus and so. By waving its arm, the simulacrum in the museum represents Humphrey as waving his arm;

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[13]Williamson, *Modal Logic as Metaphysics*, treats modal logic as a theory with metaphysical consequences, to be judged by theoretical criteria. I do not deny the importance of rigor and formalization! But given the multiplicity of mathematical models for modality, I do not think that much metaphysical weight can be derived from them. His strategy reverses what strikes me as the proper order for a philosophy of logic: from the metaphysics and semantics, to the mathematmatical structure.

> by waving his arm, or by winning the presidential election, the other-worldly Humphrey represents the this-worldly Humphrey as waving or as winning. That is how it is that Humphrey — our Humphrey — waves or wins according to the other world. This is counterpart theory, the answer I myself favour to the question how a world represents *de re*. (Lewis, *On The Plurality of Worlds*, 194)

So other realms are relevant to our world to this extent: they represent it as different than it is. Given counterpart theory, then, Lewis and the ersatzer each offer representations of actuality.

Jubien argues that "no one could bring it about that an entity represents Humphrey without being causally connected to both Humphrey and the entity" (65). If this is right, Lewis's entities do not represent actuality, and to this extent, his view is at a disadvantage relative to ersatzism. But allow that similarities are sufficient to set up something like representation even without causal interaction. Then the worlds as representers land in very much the same boat as ersatz stories. A set of sentences can represent what is not possible. This is the reason for axioms or the like to constrain stories. But so a concrete entitiy may represent what cannot be. Thus Krikpe argues that if Queen Elizabeth II is not in fact the natural daughter of the Trumans, it is metaphysically impossible for her to have been the natural daughter of the Trumans (*Naming and Necessity*, 110-13). Say this is right. Still, "this would not prevent someone from staging a play in which she was represented as being the Truman's daughter" (Jubien, 66). One might require "similarity with respect to necessary features" or otherwise build modality into the representations. But then worlds are not doing the modal work. Other worlds are not modally relevant just by standing in representation relations to actuality.

So any worlds theory must appropriately connect its worlds to what is actually possible and necessary. But, says Jubien, there is a "deep and fundamental weirdness' in the supposition that any such connection exists.

> The weirdness is this. Suppose it's necessary that all **A**s are **B**s. The central-tenet *analysis* is that in every possible world all **A**s are **B**s. So the necessity arises from what goes on in all the worlds taken together. There's nothing intrinsic to any **A**-containing world, even in all of its maximal glory, that *forces* all of its **A**s to be **B**s. It's as if it just *happens* in each such world that all of its **A**s are **B**s, that from the strictly internal point of view of any world, it's *contingent*, a mere coincidence. . . . The theory provides no basis for understanding why these contingencies repeat unremittingly across the board (while others do not). As a result,

> it provides no genuine analysis of necessity.... *Of course* if something is necessary, *and* there really are all these "possible worlds," then the something that is necessary will be true in each of them. But that doesn't tell us *why* it is true in each of them, in other words, what its necessity consists in. (74-5)

So worlds do not explain necessity. And an account of whatever intrinsic features constrain worlds to be the way they are would seem to render the worlds superfluous for an analysis of modality, as the real work is done by the intrinsic features.

Jubien himself holds that necessities are constrained by relations among platonic properties — it is from these relations that we obtain the required modal oomph. According to platonism, for something to be a horse is for it to instantiate the property BEING A HORSE, and for something to be an animal is for it to instantiate the property BEING AN ANIMAL. This requires that the properties themselves have distinct intrinsic natures. Given the natures, the reason that necessarily horses are animals is that "*the two properties' intrinsic natures together guarantee it*" (93). Thus one property may *entail* another. Similarly, horses are possibly wild if the properties BEING A HORSE and BEING WILD are *compatible* in the sense that neither entails the negation of the other. Jubien has "no opinion" about the details of property entailment (94). In particular, it is not a thesis about property constitution (though in other places he has entertained mereological hypotheses). So, for example, when something instantiates BEING A HORSE it instantiates NOT BEING A XYLOPHONE — though the one is not a constituent of the other. For Jubien's purposes, it is sufficient to think of the entailment relations as primitive. In the end, then, for "necessarily all horses are animals" and "possibly some horses are blue," Jubien gives, "necessarily anything that instantiates BEING A HORSE instantiates BEING AN ANIMAL," and "possibly something instantiates BEING A HORSE and BEING BLUE." The first is necessary insofar as BEING A HORSE entails BEING AN ANIMAL. The second is so if BEING A HORSE is compatible with BEING BLUE.

3.3.2   Worlds Again

So the charge is tht worlds, ersatz or otherwise, are superfluous for an analysis of modality. But there may yet be a role for worlds. Begin with some standard and simple analyses (compare Mondadori and Morton, "Modal Realism"). A notion of entailment or provability is familiar from logic. But on the standard account,

  PD Sentence $\mathcal{A}$ is *provable* iff there is a derivation of it.

where, in the simplest case, a *derivation* is a sequence of sentences such that each member is either an axiom or follows from previous members by a rule. Similarly, in a game of chess,

  WG White *can win* from position *p* iff some game overlapping *p* results in checkmate of black.

where a *game* is a sequence of moves respecting the rules of chess. Without an account of the derivations or games, it would be bizarre simply to point at some sequences of sentences and say, "$\mathcal{A}$ is provable iff one of these has $\mathcal{A}$ as its last element." And it would be similarly bizarre to point at some sequences of moves and say, "white can win iff one of these overlapping *p* results in checkmate of black." As such, pointing leaves it open whether the relevant sequences respect the rules, and whether they are all the sequences under the rules. It is mysterious how entities not subject to the constraints could be relevant to the analyzed notions. It is similarly inadequate merely to announce that derivations are sequences ending in provable sentences, and games sequences overlapping winnable positions. This is why foof theory is so unsatisfying. Quantifications over proofs and games become relevant precisely because of the constraints to which they are responsible. It is no accident that they are constrained in certain ways. Their "modal oomph" results from the relation to basic rules functioning as a constraint on the range of combinations. But the rules do not simply list all the entailments. Rather the full range of entailments is fixed by the rules in combination with quantification over the the derivations or games. Thus derivations and games are essential to analyses PD and WG.

　Similarly, one might think ersatz stories could be relevant to metaphysical modality. Suppose partial stories and truth in them as above. Then one might offer,

  PS Sentence $\mathcal{A}$ is *possibly true* iff a story according to which $\mathcal{A}$ is a member of some possible book.

where possible books are defined as above, and we require of the stories in possible books that they respect some basic constraints. Let the constraints be precisely the ones to which Jubien appeals. Thus stories in possible books make it true that anything that instantiates BEING A HORSE instantiates BEING AN ANIMAL, and the like. So the suggestion is that possibility is a quantification over stories; and stories are relevant insofar as they represent combinations subject to constraints grounded in the actual intrinsic natures of properties. Just as derivations and games are not superfluous to PD and WG, so worlds are not superfluous to PS. We suppose that the constraints do not include all the entailments. Rather, axioms are some basic constraints; the full range

of possibilities is fixed by the constraints only in combination with quantification over the stories. In this way, the stories remain essential to PS.

Jubien might allow the biconditional PS. He would not, however, allow it as an analysis of modality. Though worlds may represent possibilities by respecting intrinsic constraints, they are superfluous if the constraints do all the work: On his view, necessities are given by property entailment; at best, stories merely exhibit what they are. Given this, there is no work left for stories to do. However, even supposing his platonism, not all property entailments are so straightforward as in the case of BEING A HORSE and BEING AN ANIMAL, or BEING A HORSE and BEING WILD. Consider whether pigs can fly. This does not seem to be a simple matter intrinsic to BEING A PIG and BEING A FLYING THING. Rather, the possibility depends on some complex interaction between the density of pigs, the nature of gravity, the atmosphere, laws of flight and the like. One might respond that BEING A FLYING THING builds in features of gravity, the atmosphere and laws of flight, so that the relations remain intrinsic. Even so, surely entailments of BEING A FLYING THING are not independent of entailments from BEING GRAVITY, BEING AN ATMOSPHERE, and BEING THE LAWS OF FLIGHT. It is natural to think entailments of a complex BEING A FLYING THING result from entailments of constituent properties. But if this is right, there remains room for a class of "basic" entailments from which others result. Pigs cannot fly when every combination of basic properties rules it out. But with entailments of complex properties given by combinations subject to basic constraints, one might offer quantification over all stories that respect the basic constraints (or more directly, quantification over the properties themselves) precisely in order to complete the account of entailment and compatibility. In this case, even under an entailment picture, stories have a non-weird role for an account of modality.

3.4 Foundations

So the proposal is that worlds contribute to an account of modality in combination with some basic constraints derived from the nature of properties. In this section, I say a bit about how this will go.

3.4.1 Properties

In anticipation of chapter 4, consider a particularly simple theory of properties. There are some basic properties akin, perhaps, to fundamental properties of physics. Other properties are constructed from, or supervenient upon, combinations of these. A thing instantiates a conjunction of properties when it instantiates them both, a disjunction

when it instantiates one or the other, and a negation when it instantiates a complement given some background class — as a thing is not a pine when it is a cedar, an oak, or the like. Let us suppose that for any set {BEING R, BEING S...} there are properties BEING R ∧ S ∧... and BEING R ∨ S ∨.... In addition, each BEING R has a negation, BEING ¬R. Notice that for any BEING R there is BEING ¬R so that, given arbitrary conjunctions, there is BEING R ∧ ¬R. This may seem natural, if we think of properties as independent entities subject to arbitrary combinations, and so think of them along platonistic lines. Of course, some properties formed this way are not *instantiable*, but that does not make them properties any the less.

Once we say this, it is apparent that the approach to properties is incompatible with accounts on which there are no uninstantiated properties (as Armstrong, *A World of States of Affairs*). Similarly, BEING R ∧ ¬R is one thing, and BEING S ∧ ¬S is another. So the view is more fine-grained than one on which a property is just the set of its this- and other-worldly instances (as Lewis, *On The Plurality of Worlds*). For the approach to modality, however, we shall not require all or even any of the details from the present picture. For modal results, the essential requirement is that properties actually have some intrinsic natures and, one way or another, the condition for having one property "builds in" or "includes" the condition for having another.

Given this much, it is reasonable to think that, say, necessarily whatever is red is colored just because of the way BEING RED and BEING COLORED are. As we have seen, this is a theme from Jubien.

> It is a necessary truth that if something is red, then it is colored. But where does this necessity come from?... The necessity comes from the fact that part of what redness does in constituting something as being red is to constitute it as being colored. One way of thinking about this (which may or may not be the way it actually works) is to think of BEING COLORED as an integral part of BEING RED, a subcomplex. At any rate, BEING RED bears a very special relation to BEING COLORED, a relation that "supervenes" on the intrinsic properties of the two complex entities. This relation is such that anything that instantiates the first property is *thereby* compelled to instantiate the second. Let's call this special "intrinsic" relation *entailment*. (Actualism and Iterated Modalities, 119-20, notation altered; see also, *Ontology, Modality, and the Fallacy of Reference*, 111-15, along with the discussion from above.)

Jubien proposes that modal properties result from property entailments. This requires that properties have some intrinsic natures. As illustrated by the example of BEING A

HORSE and NOT BEING A XYLOPHONE, he does not not finally put this forward as a thesis about property constitution.

However, as suggested in the quoted passage, the point about property entailments is natural given something like the structured picture from above. Given the nature of BEING R ∨ S with BEING R and BEING S as parts, a thing which instantiates BEING R instantiates a part of BEING R ∨ S which is sufficient for the instantiation of that property; so a thing which instantiates BEING R thereby instantiates BEING R ∨ S. And a thing which instantiates BEING R ∨ S thereby instantiates BEING R or BEING S. Similarly a thing which instantiates BEING R ∧ S thereby instantiates BEING R and BEING S, and a thing which instantiates BEING R and BEING S thereby instantiates BEING R ∧ S. Though it is easy to speak this way, and I shall continue to do so, the point should not be vacuous in the case of uninstantiable properties; the point is rather that the conditions for the instantiation of BEING R and BEING S are the very conditions which must be met for the instantiation of BEING R ∧ S; this is why in having one, a thing thereby has the other. And similarly in other cases.

While questions about negation are bound to be controversial, our picture on which there are some basic "positive" properties, with negations formed as compliments against a background class, pushes in the direction of a picture along the following lines.
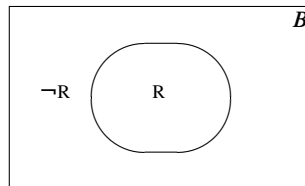
Against some background B of conditions $B_1, B_2 \ldots$ members of some collection are sufficient for BEING R and of its complement for BEING ¬R. Thus, for example, the background may be a set $S = \{s_1, s_2 \ldots\}$ of shapes, where BEING R includes ones sufficient for being round, and the negation ones in its complement — being square, triangular, and the like (possibly including conditions sufficient for being unextended). Or maybe the background is a set $C = \{c_1, c_2 \ldots\}$ of colors, BEING R ones sufficient for being red, where the negation includes being green, blue and so forth. In this case, though BEING A HORSE is no part of BEING A XYLOPHONE, against the right background class it might (notwithstanding the above suggestion) be part of NOT BEING A XYLOPHONE so that constituency would account for the entailment from BEING A HORSE to NOT BEING A XYLOPHONE. From this picture, a thing which instantiates BEING R

thereby fails to instantiate BEING ¬R; and a thing which instantiates BEING ¬R thereby fails to instantiate BEING R.

Thus, from the structured picture, we are led to a sort of "logic of analysis" including the following principles,

Dsj $C\text{PRSD} \rightarrow \Box\forall x[I\text{P}x \leftrightarrow (I\text{R}x \lor I\text{S}x)]$

Cnj $C\text{PRSC} \rightarrow \Box\forall x[I\text{P}x \leftrightarrow (I\text{R}x \land I\text{S}x)]$

Neg $C\text{PRN} \rightarrow \Box\forall x[I\text{P}x \leftrightarrow \neg I\text{R}x]$

If P is constituted disjunctively by R and S, then necessarily a thing is P iff it is R or S. If P is constituted conjunctively by R and S then necessarily a thing is P iff it is R and S. And if P is the negation of R, then necessarily a thing is P iff it is not R.[14] So if BEING COLORED is a conjunctive constituent of BEING RED or, alternatively BEING RED is a disjunctive constituent of BEING COLORED, then necessarily anything that is red is colored. The modal constraints result from actual property structures. In the end, it is not important that the basic constraints result from any particular account of *structure*, but rather that they somehow result from the way properties are. Thus this simple case is a sort of model for a larger proposal according to which the the axioms or basic constraints on worlds result from the actual intrinsic nature of properties.

The example may be extended somewhat to include the *de re* case. Constraints are similar except that from a kind alone, BEING A PARTICULAR K is left with "slots" to be filled in the individual case. We have suggested that for any kind K, BEING A K involves having certain general features and/or standing in relations to individuals of certain sorts, where BEING A PARTICULAR K requires having instances of those features and standing in those relations to particular individuals. For a simple and schematic example, from a kind K there may be an unsaturated structure BEING SOMETHING WITH F(y) AND G(y) — where the saturated essence $\langle a, \text{K} \rangle$ = BEING SOMETHING WITH F(a) AND G(a) — has its slots "plugged" from the constituting a. Then,

DR $\forall\text{R}\forall y[C\text{R}y\text{K} \rightarrow \Box\forall z(I\text{R}z \leftrightarrow (I\text{F}yz \land I\text{G}yz))]$

If R is constituted by y and K, necessarily a thing has R iff it has F to y and G to y. So from $C\text{R}a\text{K}$ it follows that $\Box\forall z(I\text{R}a \leftrightarrow (I\text{F}ay \land I\text{G}ay))$. And the idea is

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[14]The condition for negation is complicated in case the background class is less than exhaustive (so that a thing might fail to fall under either a property or its negation) or if the members of the background class are not mutually exclusive (so that a thing might fall under both). In either of these cases a complete logic of analysis is other than classical. For discussion, see the next chapter and Roy, "Making Sense of Relevant Semantics."

that principles like P1′ - P3′ on p. 78 result from the logic of analysis together with features from constituting objects.

So far, modal constraints are traced to something like property structure. This leaves it open what to say about basic, unstructured, properties. Insofar as properties are basic, they do not appear on the left side (as the ᴘ) in principles like Dsj, Cnj and Neg. So it is not clear how or whether they are constrained relative to one another. First, observe that there may be no basic properties. Perhaps any property is constituted by others which are constituted by others, without end. But we have adopted a picture which allows for basic properties, as perhaps simple properties from physics. Alternatively, in a related situation, Armstrong offers charge and mass as samples for simple properties and allows that simple states of affairs are 'Hume independent' combining in arbitrary ways (*Combinatorial Theory of Possibility*, 40-1). His view is technically attractive. But it may very well get the facts of modality wrong. Democritus, for example, would no doubt have maintained that an indivisible unit of mass is inevitably associated with a shape. And, while nobody claims the standard model of particle physics is metaphysically necessary, it is not obvious that one of the fundamental physical quantities as mass, charge or spin can appear apart from some values of the others. Following on Democritus, perhaps one day a theory of everything will reveal that a connection of this sort is necessary — that, say, charge makes no sense apart from some particular qualities as of mass or whatever.

We have already a the beginning for a suggestion of how this might be so. Thus Denkel,

> On its own, a particular property is an *unsaturated* entity; it is saturated by existing in compresence with other properties. Together, a plurality of unsaturated existences complement and saturate each other, forming a (more) saturated entity. A property is saturated *fully*, when it exists in a compresence of sufficient diversity. At such a level of sufficiency, every property in the compresence is saturated by every other fully, and the *complete* compresence they form together is a unity, an object, a fully saturated entity. (*Object and Property*, 191)

Denkel holds a view on which properties are unsaturated entities, instantiated only in fully saturated "compresences." We have suggested already that kinds are structures, unsaturated in the sense that they include slots to be plugged in particular cases. But similarly, accidents are accidents *of* something. So ʙᴇɪɴɢ ʙʟᴜᴇ is ʙᴇɪɴɢ ᴀ ʙʟᴜᴇ x and ʙᴇɪɴɢ sɪx ꜰᴇᴇᴛ ᴛᴀʟʟ is ʙᴇɪɴɢ ᴀ sɪx ꜰᴏᴏᴛ ᴛᴀʟʟ x where the nature of the accident puts constraints on the nature of the object to which it applies. So a kind property is instantiated only in conjunction with particular accidents and relations — a particular

height, a particular mother; and accidents are instantiated only in relation to particular kinds — a height in relation to a person, rather than a number. Thus there is a sort of "handshake" relation between kinds and accidents, where the one sets out a structure with places to be filled, and the other requires a structure which may itself include further accidents. In this sense, the properties seem to "saturate" one another. And it may be that a simple accident, some quantum of charge or whatever, though qualitatively basic in the sense that it does not include other properties, nevertheless participates in a complex "handshake" relation such that constraints appear between its instantiation and that of other (possibly basic) properties. Whether from property structure or such handshake relations, observe that all of our principles have the same general form: from some features of properties, or of properties and the world, follow modal constraints.

3.4.2   Primitive Modality

Suppose this much is roughly right: there are stories constrained from the actual intrinsic nature of properties. I turn now to some remarks about primitives of the resultant view. Even granting constraints from the actual intrinsic nature of properties, apart from some modal principles about properties, it far from obvious why the basic constraints are *modal*. Suppose our account of property structures. Given that P is conjunctively constituted by R and S it follows that whatever actually has P has R and S. But it looks a leap from the actual nature of P to the result that *necessarily* whatever has P has R and S. If P is conjunctively constituted by R and S and P has its structure necessarily, then any world where something has P is one where it has R and S, so necessarily a thing has P iff it has R and S. Perhaps it is natural to think of properties as necessary beings (as Plantinga, "Actualism and Possible Worlds," 262). As we have seen, Jubien maintains an entailment relation obtains as a result of the intrinsic nature of properties. Again, $\{\phi\}$ cannot remain identical to $\{\phi\}$ and be changed into $\{\phi, \{\phi\}\}$ just because being the latter is being a different set from being the former; similarly, against the background of abundant platonism, it may be difficult to understand how BEING P ∧ Q could remain identical to BEING P ∧ Q and be changed into BEING P ∧ R. Still, no matter how properties actually are, in one way or another, these are *modal* assertions about properties which thereby outrun the ground observation that properties actually are some way.

   From principles like Dsj, Cnj and Neg, modality has an actual ground in the sense that modal features covary with actual intrinsic structures of properties. But, apart from prior modal features for the properties, this does not amount to an account of the modality. The modal connection is an apparent primitive. We obtain such an

account, however, given that *the properties make the worlds*: That is, we tell stories in the actual Lagadonian language, whose vocabulary includes actual properties as predicates. Stories are constrained according to the way those properties are. It is immediate that those constraints are met in every story and so are met necessarily. And it is immediate that stories exhibit the full range of the possibilities. Similarly, the range of possibility in chess is exhibited by the range of games "made" by the rules. Thus I propose a two-step *analysis* of metaphysical modality. (i) Constraints are derived from the actual intrinsic natures of non-modal properties. The constraints, which in our sample picture of the properties, derive from property structures and maybe handshake relations, do not include every entailment or modal necessity. Rather, they are basic constraints on the model of the rules of a derivation system or the rules of chess. (ii) The full range of possibility results from quantification over combinations consistent with the basic constraints. Again, the range of possibility results from combinations on the model of provability in logic or the possibility of winning in chess. In the end, then, property entailment also results from the actual natures of properties and the way properties make the worlds. It is because worlds are constrained by the properties that necessarily whatever has BEING P has BEING Q. Thus the proposal is for a reductive analysis of modal properties. This reduction is not eliminative. There remain modal properties to which the analysis applies. The analysis is reductive in the sense that modal properties are analyzed in terms of ones which taken individually are non-modal. Modal properties just *are* quantifications over combinations constrained according to the actual intrinsic natures of non-modal properties.[15]

At the same time, I am hesitant to say that the *possibilities* just *are* certain sets of sentences. Books and stories are offered as a specimen of the combinations required for modality. States of affairs, complex properties or the like might do as well. (Of course, expressive power and maximality objections are bound to arise, but are likely to have solutions along the same lines as for stories.) The proposal is that modality results from quantifications over combinations consistent with the basic constraints. It is worth observing that the present account *explains* the success of semantics for modal logic as a straightforward model of the world structure, though the metaphysical account does not in any sense derive from the semantics for logic.

Contrast a theory according to which the possibilities are spatiotemporally and causally isolated concrete worlds, or books in a stack. There may be objections according to which one or another theory gets facts of modality wrong. This is the

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[15]Many interesting things have been said about the nature and even the possibility of analysis (recent instances are Divers, "The Analysis of Possibility and the Possibility of Analysis" and Sider, "Reductive Theories of Modality."

point of the expressive power objections. But suppose the theories agree about the modal facts, that they overlap extensionally. Then there is no such objection from one to the other. Nevertheless the theories differ about the location of primitives. None appeal to modality, but they differ in that to which they appeal. I have accepted objections according to which the stack of books and isolated concrete worlds are irrelevant to modality. However, stories, as I describe them, are not independent of actuality — and the idea is that we have primitives properly located. Of course primitives remain. However they are located so as to explain how modal properties result from the way things are. No doubt the plausibility of the account requires some prior modal understanding; but this is an epistemic point. The analysis applies so long as it properly unpacks modal properties.

Consider accounts according to which worlds are some *possible* stories or states of affairs, or according to which worlds are constrained by *necessary* axioms, or according to which necessary truths result from property *entailments*. In each case (supposing extensional overlap) I agree, but offer an account of the modality built into the account. The property entailments, necessary constraints, and possible stories result on combinations constrained according to the actual intrinsic natures of non-modal properties. Still, what is good for the goose is good for the gander. Assuming extensional overlap, the other theorists might agree with me, worlds are characterized by actual intrinsic natures of non-modal properties, but deny that I have described *modality*. According to the objection, a modal notion requires a modal primitive; what I have described is not modality but rather something that merely coincides with it. My response is the same: Primitives are located so as to expose how modal properties result from the way things are. Once we see that, say, whatever is red is colored because of the way BEING RED actually is, and that properties make the worlds for metaphysical modality, we see that metaphysical modality results from how the properties are.

3.5   Conclusion

In this concluding section, I amplify and tie together some themes from chapter 2 and chapter 3. First, a short discussion of truth *in* and *at* worlds, clarifies whether properties come out as necessary beings on this view. Second, discussion of a simple version of the modal version of the Ontological argument for the existence of a divine being is useful for exposing features of books and stories. Finally *de re* models add some detail to proposals for ordinary and proper things and worlds. The detail provides grist for further results from the view (and, potentially, a basis for objections

against it).

3.5.1 A Clarification

Given that the properties make the worlds, there is no world where something is red but not colored. So necessarily whatever is red is colored. But this does not require that BEING RED or BEING COLORED exist in every world. At one level, this is the simple observation that $\Box \forall x (I \text{R} x \rightarrow I \text{C} x)$ is true so long as in every world, any thing is either not red or colored. If there are worlds where R does not exist, however, we require that $I \text{R} x$ be *false* where R does not exist. The point is related to the distinction between truth *in* and truth *at* worlds (as Adams, "Actualism and Thisness," 22). For this, the ordinary idea is that a world *in* which there is no Socrates, and even no proposition about Socrates may nevertheless be one *at* which 'Socrates does not exist' is true. Though the proposition is not a member of the world, the world is nevertheless such as to make it true. Perhaps the initial picture is that we "observe" a world without Socrates and, from its nature, properly exclaim, 'Socrates does not exist there!'

Particularly on this picture of the matter, the in/at distinction is vigorously resisted by M. Davidson ("Transworld Identity, Singular Propositions, and Picture-Thinking"). He maintains that, from Plantinga, we have perfectly clear notions of truth and existence in worlds.

> IT Necessarily, a proposition p is *true in a world W* iff necessarily, if W is actual, then p is true.

> IE Necessarily, an object x *exists in a world W* iff necessarily, if W is actual, then x exists.[16]

Accounts of truth at worlds are driven by accounts of singular propositions. If there are not resources in a world to form a proposition, the proposition does not exist there; so if *Socrates does not exist* requires Socrates as a constituent, the proposition cannot exist in a world where Socrates does not. Truth at worlds is "rigged" on the basis of what is true in worlds to allow a sense in which the proposition is recovered. But, suggests Davidson in a short dialogue,

> You still haven't told me what this relation, *truth at* is. I understand truth-*in*. It's analyzed in terms of truth *simpliciter* and entailment. All you've told me is that there is this other relation which, if you'll permit

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[16]At the cited location (*Nature of Necessity*, 46) Plantinga offers as preliminary formulations "to say that p is true in a world W is to say that if W had been actual, p would have been true." And, "to say that an object x exists in a world W is to say that if W had been actual, x would have existed."

the colloquial speech, happens to come to your rescue whenever *I* say that a proposition *p* is true in a world *W*, and your metaphysics won't allow you to agree with me. It is like having a physical theory on which physicists agree predicts a particle will have spin. It turns out that the particle doesn't have spin. "That's OK," you say. "I have this other property, *schwinn*, and anything with spin has it, and this particle also has it. I can give you conditions under which a particle has schwinn, in fact. They will be such that any time a particle has spin, the particle will have *schwinn*; and any time the particle is predicted to have spin, but lacks it on my theory, it has *schwinn*. And, the fact that the particle has *schwinn* is good enough for the purposes of testing my theory, even if it doesn't have spin." This is not the way of true science. If "schwinn" isn't given a *reductive* analysis such that we understand what it is, simply coming up with such a predicate and claiming that the theory is safe because the particle satisfies this other predicate (via stipulation) won't save the original theory. (565)

And, says Davidson, the in/at distinction is bolstered by a false picture: it is as though it is possible to view worlds "from the outside," by a telescope as it were, and say from that perspective what is so in the the world. But there is no such perspective. All we have is the simple notion of truth in worlds.

But Davidson's worlds "picture" (if I may) exactly reverses the situation. We have accounts T1 and T2 which are effectively of truth *at* stories. So far, these do not directly address truth in. It is therefore truth in that is derivative on truth at. On a story account, truth in worlds is something like truth in fiction. But a fiction may describe situations where even the conditions for its own existence do not obtain. If Tolkien's *Lord of the Rings* were true, there would be no Tolkien, and no English language in which the story is told. For stories, it is natural to think that 'Socrates does not exist' and other expressions are true *at* a story just in case they are true in the usual way according to the story (by T2). 'Socrates does not exist' is true in a story iff "'Socrates does not exist' is true" is true at the story. That is, something is true *in* a story iff it is true according to the story that it is true. So if there is a story according to which (at which) there is neither Socrates nor the proposition 'Socrates does not exist', it may be that 'Socrates does not exist' is true at the story, but not in the story. In the ordinary case, we expect propositions true in a story to be the same as ones true at it. But in case we are willing to contemplate propositions that do not exist according to a story, the two may come apart. At this stage, I do not advocate any theory of singular propositions (though I have allowed that the Langadonian language does not have

the essences or the like to describe non-actual individuals). The point is only that the stories have the resources to allow what true in and at worlds to come apart.

Similarly, requiring that all the worlds are constructed by actual properties is not the same as requiring that actual properties exist according to all the worlds. If we tell a story according to which BEING RED does not exist, then according to that story, nothing is red. On some accounts of propositions at least, this story according to which nothing is red is not one according to which there is a proposition, 'nothing is red'. One might work up a sense of possibility that tracks truths in stories rather than truth at them. I have no objection to this (and gesture in that direction below). At this stage, the point is simply that the nature of worlds as made by the properties begins with truth at worlds, and builds its basic notion of possibility upon that.

3.5.2 An Application

Especially since a defense by Plantinga, modal versions of the ontological argument have received a great deal of attention (as *Nature of Necessity*, chapter 10; and *God, Freedom, and Evil*, section II.c). It is not my aim to add another epicycle to the many things that have been said about this reasoning. However we are in a position to illuminate some simple and natural observations about the modal ontological argument, and along the way illuminate some features of our view.

Here is a very simple version of the modal argument (essentially van Inwagen's minimal version from *Metaphysics*, 111). A being is necessary iff it exists in every possible world. Say a thing is *divine* (in at least the sense of being wondrous) iff it is both concrete and necessary. Perhaps numbers are necessary beings; but these are abstract. If something is divine, it is no mere abstract object, but both necessary and concrete. Plausibly any concrete being is essentially concrete; nothing is concrete in some worlds and abstract in others. Suppose an S5 system where each world has access to every other. Then if a divine being exists in one world, it exists as divine in all: it exists in all because necessary, and since it exists in each it remains necessary in each; it is concrete in all because essentially concrete. Reason as follows.

1. $\Diamond \exists x Dx$ Possibly there is a divine being

2. $\Box \forall x [Dx \rightarrow \Box(Ex \wedge Dx)]$ Necessarily, if something is divine, then it exists and is divine in every world

3. $\exists x Dx$ There is a divine being

By (1) there is a world w and object g such that Eg and Dg. From (2) $\forall x [Dx \rightarrow \Box(Ex \wedge Dx)]$ at w; so $Dg \rightarrow \Box(Eg \wedge Dg)$ and $\Box(Eg \wedge Dg)$

at w. So $Eg \wedge Dg$ at every world, including the actual world a; so $\exists x D x$ at the actual world a.

Plausibly, the second premise is axiomatic or results directly from axioms. It tells us something about what it is to be divine. The first premise asserts the *possibility* of a divine being. Even the "fool" who denies that there is such a being agrees that there *could* be one. Agree to this, and "presto" a little modal magic and there *is* a divine being. Such a being is at least unusual — perhaps even divine in the ordinary sense.

It is frequently observed that, by the same reasoning, from the possibility that there is no divine being, it follows that there is no divine being. Van Inwagen's example is that of a *knowno*, a person who knows that there is no such thing. If there could be a person who knows that there is no divine being, and knowledge implies truth, then it could be that there is no divine being. But then,

$1'$. $\Diamond \neg \exists x D x$ Possibly there is no divine being

2. $\Box \forall x [D x \rightarrow \Box (E x \wedge D x)]$ Necessarily, if something is divine, then it exists and is divine in every world

$3'$. $\neg \exists x D x$ There is no divine being

By $(1')$ there is a world w such that $\neg \exists x D x$. Suppose $\exists x D x$ at the actual world a; then there is an object g such that Eg and Dg at a. From (2) $\forall x [D x \rightarrow \Box (E x \wedge D x)]$ at a; so $Dg \rightarrow \Box (Eg \wedge Dg)$ and $\Box (Eg \wedge Dg)$ at a. So $Eg \wedge Dg$ at every world, including world w; so $\exists x D x$ at w. This is impossible; reject the assumption: $\neg \exists x D x$ at the actual world a.

On the face of it, this is puzzling. The fool is ready enough to grant that there can be a divine being, but no less inclined to insist that such a being might not exist. However given (2), it is inconsistent to admit them both. At this stage, one might seek some reason to assert one premise rather than the other. This is van Inwagen's strategy in *Metaphysics* (108-113). However, we are in a position to illuminate a natural sense on which (1) and $(1')$ are equally plausible and may be true at the same time.

Begin by broadening our picture to include stories, books of stories, and the *library* of metaphysically consistent books. It is a big library, the universe of all the metaphysically consistent books.
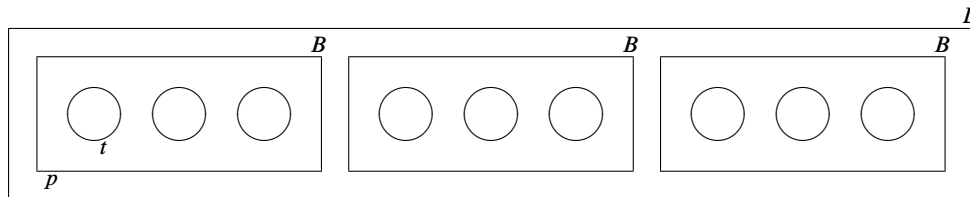
The metaphysical consistency of a book depends on α-consistency with axioms. A book is possible if it remains metaphysically consistent when augmented by any set of true stories. So the library includes many books, only some of which are possible.

Once we say this, there are at least two natural senses for the modal operators. There is [L]$\mathscr{P}$ true at a story in a book just in case $\mathscr{P}$ at every story in the library, and [B]$\mathscr{P}$ true at a story in a book just in case $\mathscr{P}$ in every story in its book. And similarly for possibility there are ⟨L⟩$\mathscr{P}$ and ⟨B⟩$\mathscr{P}$. Axioms are applied to stories across the board. Thus axioms have the relatively strong necessity [L]$\mathscr{P}$. But *de re* axioms introduce constraints on stories relative to the way things are in others. The *point* of books is to identify a range over which constraints are applied across stories. So (2) is naturally understood as [L]$\forall x[Dx \rightarrow [B](Ex \wedge Dx)]$. As such, axiomatic principles apply in all the stories in all the books. But the *de re* axiom itself says that if something is divine according to a story, then it exists and is divine according to every story in its book.

For (1) and (1′) we require few remarks about epistemology. Our metaphysics yields a few simple and obvious points about modal knowledge. On our account, possibility and necessity result from axioms. And alternate types of modality might result from alternate axiom classes. Thus, for example, if the axioms are the axioms for first order logic, the result is a logical possibility. If the axioms are logical and mathematical truths, the result is a mathematical possibility. And physical possibility may require axioms in addition to those for metaphysical modality. Insofar as we know something about the analysis of properties, we know something about the axioms. And insofar as we know about the axioms, we should be able to generate arguments to the effect that certain claims are necessarily true or necessarily false. Supposing that the axioms of logic or mathematics are included among ones for metaphysical modality, logical and mathematical necessity are sufficient but not necessary for metaphysical necessity; and logical and mathematical possibility are necessary but not sufficient for metaphysical possibility. So this is one way to access modal truths.

If we do not know all the axioms for metaphysical modality or all their consequences (and it is plausible to think that we do not) then it will not be easy to generate a direct argument to the effect that something is a possibility. In some relatively restricted context, it may be plausible to think that we can "see" that all the relevant properties do "fit" together without conflict. However, in general, it is likely that attempts to demonstrate possibility will result only in arguments from ignorance — one sees no argument for impossibility and so concludes to possibility. This is risky (and perhaps all that should be made of imaginative criteria for possibility). But there is another approach. We may reason that even if we do not know any particular axiom, we do know that no contradiction follows from the axioms and actually true sentences.

But, further, perhaps one aspect of science may be seen as involving an attempt to gauge overall consequences of the axioms for physical possibility. And, supposing the axioms for physical possibility include ones for metaphysical possibility, physical possibility is sufficient, but not necessary for metaphysical possibility; physical necessity is necessary but not sufficient for metaphysical necessity. So this is another way to access modal truths.

This leaves much unsaid. So far, however, the idea is to identify possibility by consistency. Even the inference from physical possibility is from consistency of one class of sentences to that of included sentences. ⟨B⟩ differs from ⟨L⟩ insofar as the first lets particular features of one world constrain features of others. So, insofar as the methods aim at purely general conditions, they seem aimed at ⟨L⟩$\mathscr{P}$. Grant that some such method is sufficient to motivate (1) and (1′). Then, on the natural understanding of its premises, the modal ontological argument is as follows.

1. ⟨L⟩$\exists x D x$

2. [L]$\forall x [D x \rightarrow$ [B]$(E x \wedge D x)]$

———

3. $\exists x D x$

And similarly for a version with (1′) and (3′). But these arguments are invalid. Let $\mathscr{G}$ be $\exists x D x$. Our puzzle is that, plausibly (but perhaps by an argument from ignorance) both ⟨L⟩$\mathscr{G}$ and ⟨L⟩$\neg\mathscr{G}$. Then the situation is like this.
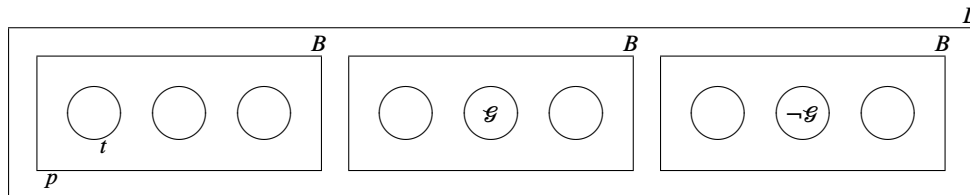
$\mathscr{G}$ and $\neg\mathscr{G}$ do not appear in the same story, and do not appear in stories in the same book. However, given ⟨L⟩$\mathscr{G}$ and ⟨L⟩$\neg\mathscr{G}$ there is a story in a book in the library where $\mathscr{G}$ and there is a story in a book in the library where $\neg\mathscr{G}$. Given (2), a book with a story where $\mathscr{G}$ has $\mathscr{G}$ in every story of its book; and a book with a story where $\neg\mathscr{G}$ has $\neg\mathscr{G}$ in every story of its book. But from these results it does not follow that $\mathscr{G}$ or $\neg\mathscr{G}$ in the actual world; for it is not yet established whether the book with $\mathscr{G}$ or the book with $\neg\mathscr{G}$ includes any actually true story. So our initial puzzle is resolved as follows: It may be that both ⟨L⟩$\mathscr{G}$ and ⟨L⟩$\neg\mathscr{G}$ are true. But they are true only insofar as the arguments from them to the conclusions that $\mathscr{G}$ and $\neg\mathscr{G}$ are invalid.

The arguments become valid if (2) is changed to (2*), $[L]\forall x[Dx \rightarrow [L](Ex \wedge Dx)]$. Of course, if a divine being exists in all the stories in all the books of a library, then it exists in true stories of the possible books. In this case, not both (1) and (1′) are true even in the sense of $\langle L \rangle \mathcal{P}$, and our original puzzle about the possibility premise has its "revenge." Observe however that (2) in the form $[L]\forall x[Dx \rightarrow [B](Ex \wedge Dx)]$ does not say anything *less* than advocates of the argument have asserted all along. If a divine being exists in some *possible* world, then it exists in every possible world and so in the actual world. The strengthened (2*) requires that something divine exist also in stories which, though metaphysically consistent, are not even possible. There are versions of the ontological argument in which the necessity associated with god is plausibly of the sort $[L]$. Somehow BEING A PERFECT BEING or or the like is such that there is such a thing in very much the way red things are colored, so that the existence of the thing is a consequence of axioms applicable in every story in every book of a library. However, the modal version of the argument with its possibility premise requires worlds constrained relative to one another. And, given the option of multiple books to measure such constraints, this comes out $[B]$, not $[L]$. From this perspective, then, (2*) is just false. On this version of the premise, then, the argument is valid, but not sound. And, so long as (2*) is not true, there remains room to assert both $\langle L \rangle \mathcal{G}$ and $\langle L \rangle \neg \mathcal{G}$.

Again, the argument is valid if the first premise is $\langle B \rangle \mathcal{G}$ in a book that includes an actually true story. Then, with $\mathcal{G}$ at some story of that book, it follows that $\mathcal{G}$ at every story of the book and so at the actually true story. Alternatively, the argument is valid if $\Diamond \mathcal{G}$, if $\mathcal{G}$ at some story of a *possible* book. But a book is possible only if it is metaphysically consistent and remains consistent upon the addition of any set of actually true stories. A metaphysically consistent book according to which Lewis is a rock is not possible insofar as a it is inconsistent with an actually true story according to which Lewis is a human. Similarly, a metaphysically consistent book with stories according to which $\mathcal{G}$ is not possible if there is an actually true story according to which $\neg \mathcal{G}$; and a metaphysically consistent book with stories according to which $\neg \mathcal{G}$ is not possible if there is an actually true story according to which $\mathcal{G}$. So $\Diamond \mathcal{G}$ requires not just that $\mathcal{G}$ be consistent, but that it be true; and $\Diamond \neg \mathcal{G}$ requires not just that $\neg \mathcal{G}$ be consistent, but that $\neg \mathcal{G}$ be true. When constraints are such that stories are not independent of actuality, *consistency is not sufficient for possibility*. Others have observed that one must agree with the conclusion of the modal ontological argument in order to admit the premise (for example Rowe, "Modal Versions of the Ontological Argument"). This may be little more than the observation that the argument is valid. However, given that consistency is not sufficient for possibility, at this stage, the point is about the nature of possibility itself: there is no independent reason to grant $\Diamond \mathcal{G}$

apart from $\mathcal{G}$, or $\Diamond\neg\mathcal{G}$ apart from $\neg\mathcal{G}$. In this case, there is no temptation to grant both $\Diamond\mathcal{G}$ and $\Diamond\neg\mathcal{G}$. And this version of the argument with $\Diamond\mathcal{G}$ as premise is question begging insofar as verification of the premise requires verification of the conclusion; and similarly for an argument with premise $\Diamond\neg\mathcal{G}$ and conclusion $\neg\mathcal{G}$.

3.5.3 Towards a Formal Model

It is possible to collect different elements of our picture into formal *de re models*. Revert to a countable language, and so set aside concerns that led to partial stories and infinitary languages. Insofar as we distinguish proper and ordinary things, the corresponding theory adapts the system MFnν from appendix D with the capacity to allow that there is no fact of the matter about proper things where they do not exist.

The vocabulary includes individual constants $a_1, a_2, a_3 \ldots$ for ordinary things; individual constants $\alpha_1, \alpha_2, \alpha_3 \ldots$ for proper things; constants $\textsc{a}_1^\textsc{n}, \textsc{a}_2^\textsc{n}, \textsc{a}_3^\textsc{n} \ldots$ for n-place relations; individual variables $x_1, x_2, x_3 \ldots$ for ordinary things; individual variables $\mathcal{x}_1, \mathcal{x}_2, \mathcal{x}_3 \ldots$ for proper things; and variables $\textsc{x}_1^\textsc{n}, \textsc{x}_2^\textsc{n}, \textsc{x}_3^\textsc{n} \ldots$ for n-place relations. Predicate symbols are $I^{n+1}, C^3, K^1, E^1, =^2, R_1^n, R_2^n, R_3^n \ldots$. The number of "places" in a relation variable, relation constant or predicate symbol is indicated by superscript, though superscripts are frequently dropped for readability. Any constant or variable is a *term*. If $\mathcal{Q}^n t_1 \ldots t_n$ is an n-place predicate symbol followed by n terms, it is a *formula*, but $I^{n+1}\textsc{r}^\textsc{n} t_1 \ldots t_n$ is well-formed iff $\textsc{r}^\textsc{n}$ is a n-place relation variable or constant and $t_1 \ldots t_n$ are n individual variables or constants. If $\mathcal{x}$ is a variable and $\mathcal{A}$ and $\mathcal{B}$ are formulas, then $\neg\mathcal{A}$, $(\mathcal{A} \wedge \mathcal{B})$, $\forall \mathcal{x}\mathcal{A}$, and $\square\mathcal{A}$ are formulas. Variables are bound and free in the usual way. $\mathcal{A}$ is a *sentence* iff it is a formula with no free variables. We allow the usual abbreviations, including $\vee$, $\rightarrow$, $\leftrightarrow$, $\exists$, and $\Diamond$.

A model includes the elements $\langle \textsf{W}, \textsf{P}, \textsf{O}, \textsf{D}_\textsf{P}, \textsf{R}, \textsf{K}, \textsf{C}_\textsf{E}, \textsf{C}_\textsf{O}, \textsf{V} \rangle$ where,

W is a set of worlds; P a set of proper things; and O a set of ordinary physical things.

$\textsf{D}_\textsf{P}$ is a function from W to subsets of P, where if $\textsf{x} \neq \textsf{y}$ and $\textsf{p} \in \textsf{D}_\textsf{P}(\textsf{x})$ then $\textsf{p} \notin \textsf{D}_\textsf{P}(\textsf{y})$. Intuitively, $\textsf{p} \in \textsf{P}$ *exists* at $\textsf{w} \in \textsf{W}$ iff $\textsf{p} \in \textsf{D}_\textsf{P}(\textsf{w})$.

R is a sequence of sets of relations where $\textsc{r}^\textsc{n} \in \textsf{Rn}$ is a (partial) function from W such that $\textsc{r}^\textsc{n}(\textsf{w}) \subseteq \textsf{D}_\textsf{P}(\textsf{w})^\textsf{n}$. In a fundamental sense we write, $\langle \textsf{p}_1 \ldots \textsf{p}_\textsf{n} \rangle \varepsilon \textsc{r}^\textsc{n}(\textsf{w})$. And $\textsf{K} \subseteq \textsf{R1}$ is the set of kind properties.

Let $\textsf{E} = \{\textsc{e} \in \textsf{R1} \mid$ for any $\textsf{w}$ in its domain $\textsc{e}(\textsf{w})$ is a unit set$\}$; and $\textsf{B} = \{\langle \textsf{p}, \textsc{k} \rangle \mid \textsf{p} \in \textsf{P}, \textsc{k} \in \textsf{K}$, and for some $\textsf{w} \in \textsf{W}, \textsf{p} \in \textsc{k}(\textsf{w})\}$. Then $\textsf{C}_\textsf{E}$ is a function

from B into E, where if $p \in D(w)$, then $p \in C_E[p, \kappa](w)$; and if $u \in C_E[p, \kappa](w)$ then $u \in \kappa(w)$.

Let h be a one-one function from essences in the range of C_E into the set O of ordinary things, and C_O be the composition of h and C_E; so $C_O[p, \kappa]$ is the member of O mapped by h from $C_E[p, \kappa]$.

So B is the set of all proper thing/kind pairs where there is a world in which the thing has the kind. These form a base for composition of ordinary essences and things. Intuitively, C_E is a map for the composition of essences; it is a function from B to E such that if p and κ are mapped to E, then p has E and anything with E has κ. And C_O is a map for composition of ordinary things, where any ordinary thing is associated with exactly one essence with the same compositional base.

Distinguish ε for proper things from a generalized $\in$ so that $\langle v_1 \ldots v_n \rangle \in R(w)$ iff $\langle f(v_1) \ldots f(v_n) \rangle \varepsilon R(w)$ where $f(v) = v$ for $v \in P$, and for $v \in O$ there are $p \in P$ and $\kappa \in K$ such that $v = C_O[p, \kappa]$ and $f(v)$ is the proper thing with $C_E[p, \kappa]$ at w. So ordinary things have just the (categorical) properties of proper things to which they are mapped by their essence function. Then $U = P \cup O$ and $D_O(w) = \{C_O[p, \kappa] \mid$ there are $p \in P$ and $\kappa \in K$ such that $C_O[p, \kappa] \in C_E[p, \kappa](w)\}$; and $D_U(w) = D_P(w) \cup D_O(w)$. $D_{Rn}(w)$ is all the members of Rn that have w in their domain; $D_R(w) = \bigcup_{n \geq 1} D_{Rn}(w)$. And $D(w) = D_U(w) \cup D_R(w)$.

V is a valuation function where,

$V[a] \in O$, $V[\alpha] \in P$, and $V[A^n] \in Rn$.

For any $\mathcal{R}^n$ and $w \in W$, $V[w, R^n] \subseteq D(w)^n$. Require that

$V[w, E] = D(w)$; so E is assigned the set of things that exist at w.

$V[w, =] = \{\langle u, u \rangle \mid u \in D(w)\}$; so $=$ is the identity predicate at w.

$V[w, K] = D_{R1}(w) \cap K$; so K is assigned the set of kind properties at w.

$V[w, C] = \{\langle C_O[p, \kappa], p, \kappa \rangle \mid \langle C_O[p, \kappa], p, \kappa \rangle \in D(w)^3\} \cup \{\langle C_E[p, \kappa], p, \kappa \rangle \mid \langle C_E[p, \kappa], p, \kappa \rangle \in D(w)^3\}$; so C says what objects form an essence or ordinary thing at a w where it is composed.

$V[w, I^{n+1}] = \{\langle R^n, v_1 \ldots v_n \rangle \mid \langle v_1 \ldots v_n \rangle \in R^n(w)\}$. So I is the instantiation predicate at w.

Set $\langle u_1 \ldots u_n \rangle \in \overline{V}[w, \mathcal{Q}^n]$ iff $\langle u_1 \ldots u_n \rangle \notin V[w, \mathcal{Q}^n]$ and for each $u_i \in P$, $u_i \in D_p(w)$. A variable designation assignment d assigns each variable x a member of O, each x a member of P, and each x^n a member of Rn; $d(x|u)$ is like d except that x is assigned to u. So V_d assigns an object to each variable and constant. For assignments to formulas,

TDR   (R) $V_d[w, \mathcal{Q}^n t_1 \ldots t_n] = 1$ iff $\langle V_d[t_1] \ldots V_d[t_n] \rangle \in V[w, \mathcal{Q}^n]$;
$\qquad V_d[w, \mathcal{Q}^n t_1 \ldots t_n] = 0$ iff $\langle V_d[t_1] \ldots V_d[t_n] \rangle \in \overline{V}[w, \mathcal{Q}^n]$.

$\quad$ ($\neg$) $V_d[w, \neg\mathcal{A}] = 1$ iff $V_d[w, \mathcal{A}] = 0$;
$\qquad V_d[w, \neg\mathcal{A}] = 0$ iff $V_d[w, \mathcal{A}] = 1$.

$\quad$ ($\wedge$) $V_d[w, \mathcal{A} \wedge \mathcal{B}] = 1$ iff $V_d[w, \mathcal{A}] = 1$ and $V_d[w, \mathcal{B}] = 1$;
$\qquad V_d[w, \mathcal{A} \wedge \mathcal{B}] = 0$ iff $V_d[w, \mathcal{A}] = 0$ or $V_d[w, \mathcal{B}] = 0$.

$\quad$ (x) $V_d[w, \forall x \mathcal{A}] = 1$ iff for any $u \in D_O(w)$, $V_{d(x|u)}[w, \mathcal{A}] = 1$;
$\qquad V_d[w, \forall x \mathcal{A}] = 0$ iff for some $u \in D_O(w)$, $V_{d(x|u)}[w, \mathcal{A}] = 0$.

$\quad$ (x) $V_d[w, \forall x \mathcal{A}] = 1$ iff for any $u \in D_P(w)$, $V_{d(x|u)}[w, \mathcal{A}] = 1$;
$\qquad V_d[w, \forall x \mathcal{A}] = 0$ iff for some $u \in D_P(w)$, $V_{d(x|u)}[w, \mathcal{A}] = 0$.

$\quad$ (x^n) $V_d[w, \forall x^n \mathcal{A}] = 1$ iff for any $R \in D_{Rn}(w)$, $V_{d(x|R)}[w, \mathcal{A}] = 1$;
$\qquad V_d[w, \forall x^n \mathcal{A}] = 0$ iff for some $R \in D_{Rn}(w)$, $V_{d(x|R)}[w, \mathcal{A}] = 0$.

$\quad$ ($\square$) $V_d[w, \square\mathcal{A}] = 1$ iff all $x \in W$ have $V_d[x, \mathcal{A}] = 1$;
$\qquad V_d[w, \square\mathcal{A}] = 0$ iff some $x \in W$ has $V_d[x, \mathcal{A}] = 0$.

These are the classical conditions for truth and falsity, and the classical conditions apply in the usual way — except that both $V_d[w, \mathcal{Q}t_1 \ldots t_n] \neq 1$ and $V_d[w, \mathcal{Q}t_1 \ldots t_n] \neq 0$ in any case where some of $V_d[t_1] \ldots V_d[t_n]$ are assigned to proper things but not in $D_p(w)$. The classical conditions are applied to expressions which are given truth values. Finally $V[w, \mathcal{P}] = 1$ iff for any d, $V_d[w, \mathcal{P}] = 1$. For a set Γ of sentences, $V[w, \Gamma] = 1$ iff for each $\mathcal{P} \in \Gamma$, $V[w, \mathcal{P}] = 1$. And $\Gamma \vDash \mathcal{P}$ iff there is no *de re* model $\langle W, P, O, D_P, R, K, C_E, C_O, V \rangle$ and $w \in W$ such that $V[w, \Gamma] = 1$ but $V[w, \mathcal{P}] \neq 1$.

So there are worlds with disjoint domains of proper things. Where t is a constant α or variable x, if $V_d[t] \notin D_P(w)$, then $V_d[w, \mathcal{Q}t] \neq 1$ and $V_d[w, \mathcal{Q}t] \neq 0$. In contrast, when a proper thing has a kind, there is a corresponding ordinary thing and essence — and function to features of the ordinary thing. Things are set up so that if t is of the sort a, A, x or x, if $V_d[t] \notin D(w)$, then $V_d[w, \mathcal{Q}t] = 0$. It remains, then, that *de re* models are for truth *at* worlds. There is, for example no problem about a world at which it is true that Socrates or some property does not exist. In some sense models

reflect *stories* about worlds. Domains of properties and individuals give the grist for an account of truth in worlds. But, to the extent that domains of propositions are allowed to vary from world to world, this account will be complicated relative to the one we have offered.

These models are sufficient to verify a modal theory including core elements from chapter 2. Let this theory include rules from NMFnν from appendix D along with the rule (Iso) to force disjoint domains for proper things. Then a theory with the following axioms is both sound and complete on *de re* models [TR7.2, 7.3].

(O1) $\forall \text{K} \forall x (\exists z C z x \text{K} \leftrightarrow K\text{K} \wedge I \text{K} x)$

(O2) $\forall \text{K} \forall x (\exists \text{Q} C \text{Q} x \text{K} \leftrightarrow K\text{K} \wedge I \text{K} x)$

(O3) $\forall x \forall \text{Q} (\exists \text{K} C \text{Q} x \text{K} \rightarrow I \text{Q} x)$

(O4) $\forall z \exists x \forall \text{R} (I \text{R} z \leftrightarrow I \text{R} x)$

(O5) $\forall z \forall \text{Q} [\exists x \exists \text{K} (C z x \text{K} \wedge C \text{Q} x \text{K}) \rightarrow \Box \forall w (I \text{Q} w \leftrightarrow \forall \text{R} (I \text{R} w \leftrightarrow I \text{R} z))]$

(O6) $\forall \text{K} \forall \text{Q} [\exists x C \text{Q} x \text{K} \rightarrow \Box \forall w (I \text{Q} w \rightarrow I \text{K} w)]$

(O7) $\forall \text{K} [K\text{K} \rightarrow \Box (E\text{K} \rightarrow K\text{K})]$

Any kind property is essentially a kind property.

(O8) $\forall \text{Q} \forall \text{R} [\exists x \exists \text{K} (C \text{Q} x \text{K} \wedge C \text{R} x \text{K}) \rightarrow \text{Q} = \text{R}]$

C is a (partial) function to essences.

(O9) $\forall \text{Q} [\exists x \exists \text{K} C \text{Q} x \text{K} \rightarrow \Box \forall u \forall v (I \text{Q} u \wedge I \text{Q} v \rightarrow u = v)]$

Necessarily, an essence is had by at most one (proper) thing.

(O10) $\forall \text{Q} \forall \text{R} \forall u \forall v [(\Diamond \exists x \exists \text{K} (C \text{Q} x \text{K} \wedge C u x \text{K}) \wedge \Diamond \exists x \exists \text{K} (C \text{R} x \text{K} \wedge C v x \text{K})) \rightarrow (\text{Q} = \text{R} \leftrightarrow u = v)]$

If Q and u are based in some x and K, and R and v are based in some x and K then Q is the same essence as R iff u is the same ordinary thing as v.

(O11) $\forall z \exists \text{Q} \Diamond \exists x \exists \text{K} (C z x \text{K} \wedge C \text{Q} x \text{K})$

For any ordinary thing z there is an essence Q such that z and Q are based in some x and K.

Among useful theorems are [TR7.1],

(Oe) $\forall z \forall_Q [\exists x \exists_K (Czx_K \wedge C_Q x_K) \rightarrow \Box \forall w (I_Q w \leftrightarrow \forall_R (I_R w \leftrightarrow I_R z))]$

(CT1) $\forall z \forall x [\exists_K C z x_K \rightarrow \forall_R (I_R x \leftrightarrow I_R z)]$

If an ordinary thing is based on a proper thing, they have all their properties in common.

(CT2) $\forall u \forall v [\exists x \exists_K (Cux_K \wedge Cvx_K) \rightarrow u = v]$

C is a (partial) function to ordinary things.

(CT3) $\forall_Q \forall z [\exists x \exists_K (C_Q x_K \wedge C z x_K) \rightarrow I_Q z]$

If Q and z are based in some x and K then z has Q.

(CT4) $\forall_Q \forall z [\exists x \exists_K (C_Q x_K \wedge C z x_K) \rightarrow \Box (\exists w I_Q w \rightarrow I_Q z)]$

If Q and z are based in some x and K then in any world where something has Q, z has Q.

(CT5) $\forall_Q \forall z [\exists x \exists_K (C_Q x_K \wedge C z x_K) \rightarrow \Box (Ez \rightarrow I_Q z)]$

If Q and z are based in some x and K then in any world where z exists, z has Q.

Thus *De re* models and the core theory illustrate relations between worlds, proper and ordinary things. They therefore develop results from the account of worlds and things as such. It is, of course, possible to supplement the theory with additional axioms along the lines of P1′-P3′ from chapter 2 along with (Dsj), (Cnj) and (Neg) from this one, to press in the direction of the more complete account of modality.

Chapter 4

Properties and Their Problems

A natural objection, made forcefully by Russell in Problems of Philosophy, is that it is a cheat to appeal to the similarity of individual qualities in an account of universals or properties. An appeal to similarity is an appeal to that which is to be explained.

This would be entirely correct if the attempt were to explain the the similarity of one quality to another. But that is not what I am doing. The attempt is to explain the existence of abstract properties. There are properties corresponding to actual qualities. Which ones? Qualities that are similar to one another.

Compare ordinary things.

Distinguish ontological order and order of explanation: we have similarity in the explanation or account of the properties. But not in their ontology. The ordinary things have properties as constituent parts. But not the proper things, and not the proper properties.

Chapter 5

Concluding and Metametaphysical Considerations

Logical Appendices

Introductory

The main text depends upon logics that develop or extend ordinary classical logic in different ways: logics are *free*, *infinitary* and *modal*. As graphically exhibited by the nineteen (!) branches of a tree diagram on the second page of Garson's excellent survey "Quantification in Modal Logic," there are many issues and strategies for, say, quantified modal logic. I develop merely some basic options. The overall project could be adapted in other directions. I assume familiarity with standard semantics and natural derivation systems for classical logic as Roy, *Symbolic Logic* and many other places.

First, for cardinal numbers μ and ν, an infinitary language $\mathcal{L}_{\mu\nu}$ allows conjunctions and disjunctions over sets of formulas of cardinality $< \mu$ and quantification over sets of variables of cardinality $< \nu$. In addition, non-logical vocabulary for a Lagadonian language appears in "ranks" corresponding to the hierarchy of things. For some operators and non-logical symbols S, then, there are the languages $\mathcal{L}_{\mu\nu}^S$; and a sentence a member of $\mathcal{L}$ iff it is a member of some $\mathcal{L}_{\mu\nu}^S$. Semantics are for languages of this sort. However, I limit derivations to fragments with maybe uncountable vocabulary, but sentences of finite length (of the sort $\mathcal{L}_{\omega\omega}^S$) — these are complete in the usual way and, in the end, fragments to which derivations are usefully applied.

An interpretation for normal sentential modal logics is a triple $\langle W, A, V \rangle$ where W is a set of worlds, A is a binary *access* relation on worlds, and V is a valuation that assigns truth values to atomic sentences at worlds. So, where $\mathcal{A}$ is atomic and $w \in W$, $V[w, \mathcal{A}] = 1$ or $V[w, \mathcal{A}] = 0$. When one moves from ordinary sentential logic to quantified logic, one moves from a simple interpretation which assigns a truth value to atomic sentences, to interpretations which include a *universe* of objects, with assignments to constants and predicate symbols. It is natural to think we could do something similar in the transition from sentential to quantified modal logic. Thus, for example, we might say an interpretation is $\langle W, U, D, A, V \rangle$ where W is a set of worlds, U a set of objects, D a function from W to subsets of U, A a binary access relation on worlds, and V a function which assigns a member of U to each constant

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symbol, and a subset of U^n to each n-place predicate symbol at each world. Then, intuitively, for w ∈ W, D(w) says which things exist in world w. And V says which things are assigned to constants and assigned to predicate symbols at worlds. Thus, we might have $V[b]$ = Bill, $V[w, H^1]$ = {Bill, Hill} and $V[x, H^1]$ = {Hill, Jill}; so that Hb turns out true at w but false at x. And $\Box Hb$ is true at x just in case Hb at every world z such that xAz.

But it is natural to think that Bill does not exist at every world — that D varies from one world to the next. And it is natural to think that 'everybody is happy' should come out true at w just when all the people *at* w are happy, and 'somebody is happy' should come out true just when someone at w is happy. For this, for evaluation at w, quantifiers need to be restricted to the members of D(w). So far, so good. But $\exists x(x = b)$ is a theorem of ordinary predicate logic; so that, if theorems of logic are necessary truths, Bill turns out to be a necessary being. Theological concerns to the side, something seems to have gone awry: for we began with precisely the assumption that Bill does *not* exist at every world.

Though its original motivation is not from possible worlds, quantified *free* logic is designed to accommodate interpretations with a universe U of objects greater than the domain D over which quantifiers range. The idea seems to have been that there are objects which do not exist (Pegasus, or the like). Whatever sense is to be made of this, from our assumptions, there would seem to be a straightforward application to the modal context, where Bill is a member of some, but not every D. To accommodate this sort of thing, relative to the classical case, free logic imposes constraints on the quantifier rules. We may thus introduce a predicate E for existence, with quantifier rules as follows. $\mathcal{P}^x_a$ is $\mathcal{P}$ with each free instance of a variable x replaced by a constant a.
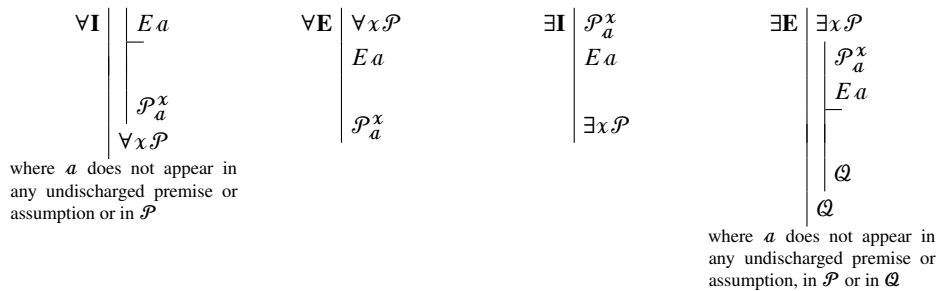
So the quantifiers operate on things that exist. As in below, a standard natural derivation version of free logic merges these quantifier rules with natural derivation rules from classical symbolic logic with equality.

We have said that $\square Hb$ is true at w just in case Bill is in the extension of H at every world accessible to w. But Bill does not exist at every accessible world. This raises the question how we are to evaluate Hb at worlds where Bill does not exist. Here are three responses: (1) A standard response, perhaps because it is technically straightforward, is to say that non-existence at a world need not prevent a thing's being in the extension of a predicate there – to let V[w, H] be any subset of U. Another reaction is to deny that an object can be in the extension of predicates at a world where it does not exist — this is to restrict V[w, H] to subsets of D(w). So (2) we might let Hb be *false* at worlds where Bill does not exist. Alternatively, (3) we might say Hb is neither true nor false at worlds where Bill does not exist.[1] In the appendices that follow, I develop systems compatible with each of these options.

---

[1]Options (2) and (3) seem compatible with "serious actualism" as defended by Alvin Plantinga, though (2) is like the one he explicitly endorses (see, for example, Actualism and Possible Worlds).

Appendix A

Infinitary Free Logic

In this appendix, I develop a simple infinitary free logic IF1 where quantification is compatible with the option (1) above — so that an object may be in the extension of a predicate, whether or not it is among the things that exist. Other options are explored in the next appendices. Soundness and completeness for derivations are demonstrated in [TR1.1, 1.2].

A.1 Language / Semantic Notions

LIF1 For some $\mathcal{L}^S_{\mu\nu}$, the *vocabulary* consists of a set of variables x_η, a set of constants c_η, a set of predicate symbols E, $=$, R^n_η and operator symbols $\neg$, $\bigwedge$, $\forall$. The number of "places" in a predicate symbol is indicated by superscript where '$=$' is always two-place and 'E' one-place. Any variable or constant is a *term*. If $Q^n t_1 \ldots t_n$ is an n-place predicate symbol followed by n terms, it is a *formula*. If $\mathcal{P}$ is a formula, Π is a set of formulas with $|\Pi| < \mu$, Ξ is a set of variables with $|\Xi| < \nu$, then $\neg\mathcal{P}$, $\bigwedge\Pi$, and $\forall\Xi\mathcal{P}$ are *formulas*. Variables are bound and free in the usual way. $\mathcal{P}$ is a *sentence* iff it is a formula with no free variables. Allow the usual abbreviations. In particular, $\bigvee\Pi$ is $\neg\bigwedge\{\neg\mathcal{P} \mid \mathcal{P} \in \Pi\}$; $\exists\Xi\mathcal{P}$ is $\neg\forall\Xi\neg\mathcal{P}$; $\mathcal{P} \wedge \mathcal{Q}$ is $\bigwedge\{\mathcal{P}, \mathcal{Q}\}$; $\forall x\mathcal{P}$ is $\forall\{x\}\mathcal{P}$; and similarly for other operators including $\vee$, $\rightarrow$, and $\leftrightarrow$.

IIF1 An *interpretation* for $\mathcal{L}^S_{\mu\nu}$ is $\langle \mathsf{U}, \mathsf{D}, \mathsf{V} \rangle$ where U is a set of objects and D a subset of U. V is a function such that for any constant c, $\mathsf{V}[c] \in \mathsf{U}$; and for any n-place predicate symbol Q^n, $\mathsf{V}[Q^n] \subseteq \mathsf{U}^n$. Require that $\mathsf{V}[E] = \mathsf{D}$; and $\mathsf{V}[=] = \{\langle \mathsf{u}, \mathsf{u} \rangle \mid \mathsf{u} \in \mathsf{U}\}$.

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A variable designation assignment d assigns each variable a member of U. Where Ξ is a set of variables and f is a function from Ξ to U, $d(\Xi|f)$ is like d except that the assignment to any $x \in \Xi$ is $f(x)$. So V_d, that is V together with d, assigns a member of U to each term.

TIF1 For assignments to formulas,

(R) $V_d[\mathcal{Q}^n t_1 \ldots t_n] = 1$ if $\langle V_d[t_1] \ldots V_d[t_n] \rangle \in V_d[\mathcal{Q}^n]$, and 0 otherwise.

($\neg$) $V_d[\neg \mathcal{A}] = 1$ if $V_d[\mathcal{A}] = 0$, and 0 otherwise.

($\wedge$) $V_d[\bigwedge \Pi] = 1$ if $V_d[\mathcal{A}] = 1$ for each $\mathcal{A} \in \Pi$, and 0 otherwise.

($\forall$) $V_d[\forall \Xi \mathcal{A}] = 1$ if for any f from Ξ to D, $V_{d(\Xi|f)}[\mathcal{A}] = 1$, and 0 otherwise.

$V[\mathcal{A}] = 1$ iff for any d, $V_d[\mathcal{A}] = 1$. And $V[\Gamma] = 1$ iff for each $\mathcal{A} \in \Gamma$, $V[\mathcal{A}] = 1$. Then, where the members of Γ and $\mathcal{A}$ are sentences of $\mathcal{L}^S_{\mu\nu}$,

VIF1 $\Gamma \vDash_{\text{IF1}} \mathcal{A}$ iff there is no IF1 interpretation $\langle U, D, V \rangle$ for $\mathcal{L}^S_{\mu\nu}$ such that $V[\Gamma] = 1$ and $V[\mathcal{A}] = 0$.

A.2 Natural Derivations: NF1

As suggested above, rules for an $\mathcal{L}^S_{\omega\omega}$ combine ordinary classical logic with restricted quantifier rules. Rules for $\vee$, $\rightarrow$, $\leftrightarrow$ and $\exists$ are derived.

$$\lor\text{I} \quad \begin{array}{|l} \mathcal{P} \\ \\ \mathcal{P} \lor \mathcal{Q} \end{array} \qquad\qquad \lor\text{I} \quad \begin{array}{|l} \mathcal{P} \\ \\ \mathcal{Q} \lor \mathcal{P} \end{array} \qquad\qquad \lor\text{E} \quad \begin{array}{|l} \mathcal{P} \lor \mathcal{Q} \\ \quad\begin{array}{|l}\mathcal{P}\\ \\ \mathcal{R}\end{array} \\ \quad\begin{array}{|l}\mathcal{Q}\\ \\ \mathcal{R}\end{array} \\ \mathcal{R} \end{array}$$

$$\to\text{I} \quad \begin{array}{|l} \begin{array}{|l}\mathcal{P}\\ \\ \mathcal{Q}\end{array} \\ \mathcal{P} \to \mathcal{Q} \end{array} \qquad\qquad \to\text{E} \quad \begin{array}{|l} \mathcal{P} \to \mathcal{Q} \\ \mathcal{P} \\ \\ \mathcal{Q} \end{array}$$

$$\leftrightarrow\text{I} \quad \begin{array}{|l} \begin{array}{|l}\mathcal{P}\\ \\ \mathcal{Q}\end{array} \\ \begin{array}{|l}\mathcal{Q}\\ \\ \mathcal{P}\end{array} \\ \mathcal{P} \leftrightarrow \mathcal{Q} \end{array} \qquad \leftrightarrow\text{E} \quad \begin{array}{|l} \mathcal{P} \leftrightarrow \mathcal{Q} \\ \mathcal{P} \\ \\ \mathcal{Q} \end{array} \qquad \leftrightarrow\text{E} \quad \begin{array}{|l} \mathcal{P} \leftrightarrow \mathcal{Q} \\ \mathcal{Q} \\ \\ \mathcal{P} \end{array}$$

$$\forall\text{I} \quad \begin{array}{|l} \begin{array}{|l}E\,a\\ \\ \mathcal{P}^x_a\end{array} \\ \forall x \mathcal{P} \end{array} \qquad \forall\text{E} \quad \begin{array}{|l} \forall x \mathcal{P} \\ E\,a \\ \\ \mathcal{P}^x_a \end{array} \qquad \exists\text{I} \quad \begin{array}{|l} \mathcal{P}^x_a \\ E\,a \\ \\ \exists x \mathcal{P} \end{array} \qquad \exists\text{E} \quad \begin{array}{|l} \exists x \mathcal{P} \\ \quad\begin{array}{|l}\mathcal{P}^x_a\\ E\,a\\ \\ \mathcal{Q}\end{array} \\ \mathcal{Q} \end{array}$$

where a does not appear in
any undischarged premise or
assumption or in $\mathcal{P}$

where a does not appear in
any undischarged premise or
assumption, in $\mathcal{P}$ or in $\mathcal{Q}$

$$=\text{I} \quad \begin{array}{|l} \\ \\ a = a \end{array} \qquad\qquad =\text{E} \quad \begin{array}{|l} a = c \\ \mathcal{P} \\ \\ \mathcal{P}\,^a/_c \end{array} \quad \begin{array}{|l} c = a \\ \mathcal{P} \\ \\ \mathcal{P}\,^a/_c \end{array}$$

one or more instances of a replaced by c

Then,

NF1 $\Gamma \vdash_{\text{NF1}} \mathcal{A}$ iff there is an NF1 derivation of $\mathcal{A}$ from the members of Γ.

Appendix B

Infinitary Free Modal Logic

We now develop the logic from appendix A to a modal logic, retaining the first option on which things may be in the extension of a predicate at a world where they do not exist. For soundness and completeness see [TR2.1, 2.2].

B.1 Language / Semantic Notions (IF1α)

LIF1α For a modal language $\mathcal{L}^S_{\mu\nu}$, again the *vocabulary* consists of a set of variables x_η, a set of constants c_η, a set of predicate symbols E, $=$, R^n_η and operator symbols $\neg$, $\bigwedge$, $\forall$ this time with $\square$. The number of "places" in a predicate symbol is indicated by superscript where '$=$' is always two-place and 'E' one-place. Any variable or constant is a *term*. If $Q^n t_1 \ldots t_n$ is a n-place predicate symbol followed by n terms, it is a *formula*. If $\mathcal{P}$ is a formula, Π is a set of formulas with $|\Pi| < \mu$, Ξ is a set of variables with $|\Xi| < \nu$, then $\neg\mathcal{P}$, $\square\mathcal{P}$, $\bigwedge\Pi$, and $\forall\Xi\mathcal{P}$ are *formulas*. Variables are bound and free in the usual way. $\mathcal{P}$ is a *sentence* iff it is a formula with no free variables. Allow abbreviations as before, now including $\diamond$.

IIF1α An *interpretation* is $\langle W, U, D, A, V \rangle$ where W is a set of worlds, U a set of objects, D a function from W to subsets of U, A a subset of W^2, and V a function such that for any constant c, $V[c] \in U$ and for any n-place predicate symbol Q^n and $w \in W$, $V[w, Q^n] \subseteq U^n$. Require that $V[w, E] = D(w)$; and $V[w, =] = \{\langle u, u \rangle \mid u \in U\}$. In addition, where α is empty or some combination of the following,

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ρ	for all x, xAx	reflexivity
σ	for all x, y, if xAy then yAx	symmetry
τ	for all x, y, z, if xAy and yAz then xAz	transitivity
υ	for all x, y, xAy	universality

$\langle W, U, D, A, V \rangle$ is an IF1α interpretation when A satisfies constraints from α. It will turn out that IF1$\rho\sigma\tau$ has all the same consequences as IF1υ.

A variable designation assignment d assigns each variable a member of U. Where Ξ is a set of variables and f is a function from Ξ to U, $d(\Xi|f)$ is like d except that the assignment to any $x \in \Xi$ is $f(x)$.

TIF1α For assignments to formulas,

> (R) $V_d[w, \mathcal{Q}^n t_1 \ldots t_n] = 1$ if $\langle V_d[t_1] \ldots V_d[t_n] \rangle \in V_d[w, \mathcal{Q}^n]$, and 0 otherwise.
>
> ($\neg$) $V_d[w, \neg\mathcal{A}] = 1$ if $V_d[w, \mathcal{A}] = 0$, and 0 otherwise.
>
> ($\bigwedge$) $V_d[w, \bigwedge\Pi] = 1$ if $V_d[w, \mathcal{A}] = 1$ for each $\mathcal{A} \in \Pi$, and 0 otherwise.
>
> ($\forall$) $V_d[w, \forall\Xi\mathcal{A}] = 1$ if for any f from Ξ to D(w), $V_{d(\Xi|f)}[w, \mathcal{A}] = 1$, and 0 otherwise.
>
> ($\square$) $V_d[w, \square\mathcal{A}] = 1$ if all $x \in W$ such that wAx have $V_d[x, \mathcal{A}] = 1$, and 0 otherwise.

$V[w, \mathcal{A}] = 1$ iff for any d, $V_d[w, \mathcal{A}] = 1$. And $V[w, \Gamma] = 1$ iff for each $\mathcal{A} \in \Gamma$, $V[w, \mathcal{A}] = 1$. Then, where the members of Γ and $\mathcal{A}$ are sentences of $\mathcal{L}_{\mu\nu}^S$,

VIF1α $\Gamma \models_{\mathbb{IF}1\alpha} \mathcal{A}$ iff there is no IF1α interpretation $\langle W, U, D, A, V \rangle$ for $\mathcal{L}_{\mu\nu}^S$, and $w \in W$ such that $V[w, \Gamma] = 1$ and $V[w, \mathcal{A}] = 0$.

B.2   Natural Derivations: NF1α

Where *s* is any integer, let $\mathcal{A}_s$ be a *subscripted formula*. For subscripts *s* and *t*, allow expressions of the sort *s.t*. Intuitively, subscripts indicate worlds where $\mathcal{A}$ is true or false, and *s.t* just in case world *s* has access to world *t*. Derivation rules apply to these expressions. Begin with a natural extension of rules from before. Modal rules are new, with rules for $\diamond$ derived.

$$\mathbf{R} \;\middle|\; \mathcal{P}_s \\[2em] \mathcal{P}_s$$

$$\neg\mathbf{I} \;\middle|\middle|\; \underline{\mathcal{P}_s} \\[1.5em] \mathcal{Q}_t \\ \neg\mathcal{Q}_t \\ \neg\mathcal{P}_s$$

$$\neg\mathbf{E} \;\middle|\middle|\; \underline{\neg\mathcal{P}_s} \\[1.5em] \mathcal{Q}_t \\ \neg\mathcal{Q}_t \\ \mathcal{P}_s$$

$$\wedge\mathbf{I} \;\middle|\; \mathcal{P}_s \\ \mathcal{Q}_s \\[1.5em] (\mathcal{P} \wedge \mathcal{Q})_s$$

$$\wedge\mathbf{E} \;\middle|\; (\mathcal{P} \wedge \mathcal{Q})_s \\[1.5em] \mathcal{P}_s$$

$$\wedge\mathbf{E} \;\middle|\; (\mathcal{P} \wedge \mathcal{Q})_s \\[1.5em] \mathcal{Q}_s$$

$$\vee\mathbf{I} \;\middle|\; \mathcal{P}_s \\[1.5em] (\mathcal{P} \vee \mathcal{Q})_s$$

$$\vee\mathbf{I} \;\middle|\; \mathcal{P}_s \\[1.5em] (\mathcal{Q} \vee \mathcal{P})_s$$

$$\vee\mathbf{E} \;\middle|\; (\mathcal{P} \vee \mathcal{Q})_s \\ \quad\underline{\mathcal{P}_s} \\[1.5em] \quad\mathcal{R}_t \\[1em] \quad\underline{\mathcal{Q}_s} \\[1.5em] \quad\mathcal{R}_t \\ \mathcal{R}_t$$

$$\rightarrow\mathbf{I} \;\middle|\middle|\; \underline{\mathcal{P}_s} \\[2em] \mathcal{Q}_s \\ (\mathcal{P} \rightarrow \mathcal{Q})_s$$

$$\rightarrow\mathbf{E} \;\middle|\; (\mathcal{P} \rightarrow \mathcal{Q})_s \\ \mathcal{P}_s \\[1.5em] \mathcal{Q}_s$$

$$\leftrightarrow\mathbf{I} \;\middle|\middle|\; \underline{\mathcal{P}_s} \\[2em] \mathcal{Q}_s \\[1em] \underline{\mathcal{Q}_s} \\[2em] \mathcal{P}_s \\ (\mathcal{P} \leftrightarrow \mathcal{Q})_s$$

$$\leftrightarrow\mathbf{E} \;\middle|\; (\mathcal{P} \leftrightarrow \mathcal{Q})_s \\ \mathcal{P}_s \\[1.5em] \mathcal{Q}_s$$

$$\leftrightarrow\mathbf{E} \;\middle|\; (\mathcal{P} \leftrightarrow \mathcal{Q})_s \\ \mathcal{Q}_s \\[1.5em] \mathcal{P}_s$$

$$\forall\mathbf{I} \;\middle|\middle|\; E\,a_s \\[2em] (\mathcal{P}^x_a)_s \\ \forall x \mathcal{P}_s$$

where a does not appear in any undischarged premise or assumption or in $\mathcal{P}$

$$\forall\mathbf{E} \;\middle|\; \forall x \mathcal{P}_s \\ E\,a_s \\[1.5em] (\mathcal{P}^x_a)_s$$

$$\exists\mathbf{I} \;\middle|\; (\mathcal{P}^x_a)_s \\ E\,a_s \\[1.5em] \exists x \mathcal{P}_s$$

$$\exists\mathbf{E} \;\middle|\; \exists x \mathcal{P}_s \\ \quad\underline{(\mathcal{P}^x_a)_s} \\ \quad\underline{E\,a_s} \\[1.5em] \quad\mathcal{Q}_u \\ \mathcal{Q}_u$$

where a does not appear in any undischarged premise or assumption, in $\mathcal{P}$ or in $\mathcal{Q}$

$\Box$**I** $\quad\Big|$ $\quad\Big|$ $s.t$ $\qquad$ $\Box$**E** $\Big|$ $\Box\mathscr{P}_s$ $\qquad$ $\Diamond$**I** $\Big|$ $\mathscr{P}_t$ $\qquad$ $\Diamond$**E** $\Big|$ $\Diamond\mathscr{P}_s$

$\qquad\qquad\qquad\qquad\qquad s.t \qquad\qquad\qquad\qquad s.t \qquad\qquad\qquad\qquad\qquad s.t$

$\qquad\qquad\qquad\quad \mathscr{P}_t \qquad\qquad\qquad\quad \mathscr{P}_t \qquad\qquad\qquad\quad \Diamond\mathscr{P}_s \qquad\qquad\qquad\qquad \mathscr{P}_t$

$\qquad\qquad\qquad\quad \Box\mathscr{P}_s$

where t does not appear in
any undischarged premise or
assumption

$\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad \mathcal{Q}_u$

$\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad \mathcal{Q}_u$

where t does not appear in
any undischarged premise or
assumption and is not u

AMρ $\quad\Big|$ $\qquad\qquad$ **AM**σ $\Big|$ $s.t$ $\qquad\qquad$ **AM**τ $\Big|$ $s.t$ $\qquad\qquad$ **AM**υ $\Big|$

$\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad t.u$

$s.s \qquad\qquad\qquad\qquad\qquad t.s \qquad\qquad\qquad\qquad s.u \qquad\qquad\qquad\qquad s.t$

=I $\quad\Big|$ $\qquad\qquad\qquad\qquad\qquad$ **=E** $\Big|$ $(a = c)_s \qquad\quad (c = a)_s$

$\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\quad \mathscr{P}_t \qquad\qquad\quad \mathscr{P}_t$

$(a = a)_s \qquad\qquad\qquad\qquad\qquad\qquad (\mathscr{P}^a/_c)_t \qquad\quad (\mathscr{P}^a/_c)_t$

one or more instances of a replaced by c

Every subscript is 0, appears in a premise, or in the t place of an assumption for $\Box$I
or $\Diamond$E. Systems vary according to rules in the penultimate row. Where the members
of Γ and $\mathcal{A}$ are sentences without subscripts, let Γ_0 be the members of Γ, each with
subscript 0. Then,

NFnα $\quad$ $\Gamma \vdash_{\text{NF1}\alpha} \mathcal{A}$ iff there is an NF1α derivation of $\mathcal{A}_0$ from the members of Γ_0.

We get the different derivation systems insofar as AM rules may differ. Rules for the
υ system are capable of simplification; however this formulation maintains contact
with other modal systems. Here are a couple of examples.

$\vdash_{\text{NF1}\sigma\tau} \Diamond P \to \Box\Diamond P$

1	$\Diamond P_0$	A $(g, \to\text{I})$
2	0.1	A $(g, 1 \Diamond\text{E})$
3	P_1	
4	0.2	A $(g, \Box\text{I})$
5	2.0	4 AMσ
6	2.1	5,2 AMτ
7	$\Diamond P_2$	3,6 $\Diamond$I
8	$\Box\Diamond P_0$	4-7 $\Box$I
9	$\Box\Diamond P_0$	1,2-8 $\Diamond$E
10	$(\Diamond P \to \Box\Diamond P)_0$	1-9 $\to$I

$\exists y\Diamond\neg Hy \vdash_{\text{NF1}\alpha} \neg\forall x\Box Hx$

1	$\exists y\Diamond\neg Hy_0$	P
2	$\Diamond\neg Ha_0$	A $(g, 1 \exists\text{E})$
3	Ea_0	
4	0.1	A $(g, 2 \Diamond\text{E})$
5	$\neg Ha_1$	
6	$\forall x\Box Hx_0$	A $(c, \neg\text{I})$
7	$\Box Ha_0$	6,3 $\forall$E
8	Ha_1	7,4 $\Box$E
9	$\neg Ha_1$	5 R
10	$\neg\forall x\Box Hx_0$	6-19 $\neg$I
11	$\neg\forall x\Box Hx_0$	2,4-10 $\Diamond$E
12	$\neg\forall x\Box Hx_0$	1,2-11 $\exists$E

Appendix C

Quantification Over Properties

In the introduction to these appendices, we noted the options that (1) Hb is either true or false where Bill does not exist; (2) Hb is false where Bill does not exist; and (3) Hb is neither true nor false where Bill does not exist. Appendix B develops a modal logic for the relatively simple option (1). But we have observed that there may be models where the domain of properties varies as well as the domain of individuals. In this appendix, I extend the logic to include free quantification over properties. In this case, I develop option (2), and so require that expressions be false where individuals or properties do not exist.

There is a distinction between "standard" and "Henkin" models for second order logic. On standard models, n-place relations are all the members of the powerset of the domain D^n. On this option, there is a categorical second order arithmetic; more arguments are semantically valid, but derivation systems are incomplete. On Henkin models, n-place relations are some subsets of D^n. Then there is no categorical arithmetic; fewer arguments are semantically valid; but derivation systems are complete. A free second order logic supposes the Henkin-style variable domains. In this case, the second order logic is equivalent to a many sorted first order logic with a special instantiantion predicate. Insofar as the logical situation remains more clear, I adopt this latter course. For discusssion see Enderton, A Mathematical Introduction to Logic and Shapiro, Foundations Without Foundationalism. For soundness and completeness see [TR3.1, 3.2].

C.1   Language / Semantic Notions

LMIF2α For a language $\mathcal{L}^S_{\mu\nu}$, the *vocabulary* consists of a set of individual variables x_η; a set of relation variables x^n_η; a set of individual constants c_η; a set

of relation constants A_η^n; a set of predicate symbols, I^{n+1}, E, $=$, R_η^n; and operators $\neg$, $\bigwedge$, $\forall$, $\square$. The number of "places" in a relation variable, relation constant or predicate symbol is indicated by superscript, where '$=$' is always two-place, and 'E' one-place. Any variable or constant is a *term*. If $\mathcal{Q}^n t_1 \dots t_n$ is an n-place predicate symbol followed by n terms, it is a *formula*, but $I^{n+1} R^n t_1 \dots t_n$ is well-formed iff R^n is a n-place relation variable or constant and $t_1 \dots t_n$ are n individual terms.

If $\mathcal{P}$ is a formula, Π is a set of formulas with $|\Pi| < \mu$, Ξ is a set of variables with $|\Xi| < \nu$, then $\neg\mathcal{P}$, $\square\mathcal{P}$, $\bigwedge\Pi$, and $\forall\Xi\mathcal{P}$ are *formulas*. Variables are bound and free in the usual way. $\mathcal{P}$ is a *sentence* iff it is a formula with no free variables. Allow abbreviations as before.

IMIF2α An *interpretation* is $\langle W, U, R, D, A, V \rangle$ where W is a set of worlds; U is a set of objects; R is a sequence of sets of relations, such that $R \in Rn$ is a (partial) function from W to subsets of U^n; D is a function from W to sets of objects and relations from U and the union of R; and A is a subset of W^2. Then $D_U(w) = D(w) \cap U$, $D_R(w) = D(w) \cap \bigcup R$ and $D_{Rn}(w) = D(w) \cap Rn$. Require that the domain of any $R \in Rn$ is $\{w \mid R \in D(w)\}$, and for any world in its domain, $R^n(w) \subseteq D_U(w)^n$. V is a function such that for any individual constant a, $V[a] \in U$; for any relation constant A^n, $V[A^n] \in Rn$; and for any predicate symbol, $\mathcal{R}^n$, and $w \in W$, $V[w, \mathcal{R}^n] \subseteq D(w)^n$. Require that $V[w, E] = D(w)$; $V[w, =] = \{\langle u, u \rangle \mid u \in D(w)\}$; and $V[w, I^{n+1}] = \{\langle R^n, u_1 \dots u_n \rangle \mid \langle u_1 \dots u_n \rangle \in R^n(w)\}$. In addition, where α is empty or some combination of the following,

ρ	for all x, xAx	reflexivity
σ	for all x, y, if xAy then yAx	symmetry
τ	for all x, y, z, if xAy and yAz then xAz	transitivity
υ	for all x, y, xAy	universality

$\langle W, U, R, D, A, V \rangle$ is a MF2α interpretation when A satisfies constraints from α. A variable designation assignment d assigns each individual variable x a member of U, and each relation variable x^n a member of Rn. Where Ξ is a set of variables and f is a function that takes any $x \in \Xi$ to a member of U, and any $x^n \in \Xi$ to a member of Rn, $d(\Xi|f)$ is like d except that the assignment to any $x \in \Xi$ is $f(x)$. Then,

TMIF2 For assignments to formulas,

(R) $V_d[w, \mathcal{Q}^n t_1 \dots t_n] = 1$ if $\langle V_d[t_1] \dots V_d[t_n] \rangle \in V_d[w, \mathcal{Q}^n]$, and 0 otherwise.

($\neg$) $V_d[w, \neg\mathcal{A}] = 1$ if $V_d[w, \mathcal{A}] = 0$, and 0 otherwise.

($\bigwedge$) $V_d[w, \bigwedge\Pi] = 1$ if $V_d[w, \mathcal{A}] = 1$ for each $\mathcal{A} \in \Pi$, and 0 otherwise.

($\forall$) $V_d[w, \forall\Xi\mathcal{A}] = 1$ iff for any f from Ξ to $D(w)$, $V_{d(\Xi|f)}[w, \mathcal{A}] = 1$, and 0 otherwise.

($\square$) $V_d[w, \square\mathcal{A}] = 1$ if all $x \in W$ such that wAx have $V_d[x, \mathcal{A}] = 1$, and 0 otherwise.

$V[w, \mathcal{A}] = 1$ iff for any d, $V_d[w, \mathcal{A}] = 1$. And $V[w, \Gamma] = 1$ iff for each $\mathcal{A} \in \Gamma$, $V[w, \mathcal{A}] = 1$. Then, where the members of Γ and $\mathcal{A}$ are sentences of $\mathcal{L}^S_{\mu\nu}$,

VMIF2α $\Gamma \vDash_{\text{MIF2}\alpha} \mathcal{A}$ iff there is no MIF2α interpretation $\langle W, U, R, D, A, V\rangle$ for $\mathcal{L}^S_{\mu\nu}$ and $w \in W$ such that $V[w, \Gamma] = 1$ and $V[w, \mathcal{A}] = 0$.

The result is classical, with the constraint that $V_d[w, \mathcal{Q}t_1 \ldots t_n] = 0$ in any case where some of $V_d[t_1] \ldots V_d[t_n]$ are not in $D(w)$. This is option (2).

C.2   Natural Derivations: NMF2α

Rules are as in appendix B, with the requirement that x and a are of the same sort (including superscripts) for quantifier rules, and the following to replace the last row. For any atomic $\mathcal{Q}a_1 \ldots a_n$ let $P_2[\mathcal{Q}a_1 \ldots a_n]_s = (E\,a_1 \wedge \ldots \wedge E\,a_n)_s$.

P₂I	$(\mathcal{Q}a_1 \ldots a_n)_s$	**=I**	$E\,a_s$	**=E**	$(a = c)_s$		$(c = a)_s$
					$\mathcal{P}_t$		$\mathcal{P}_t$
	$P2[\mathcal{Q}a_1 \ldots a_n]_s$		$(a = a)_s$		$(\mathcal{P}^a/_c)_t$		$(\mathcal{P}^a/_c)_t$

one or more instances of a replaced by c

P_2I enforces option (2). Where the members of Γ and $\mathcal{A}$ are sentences without subscripts, let Γ_0 be the members of Γ, each with subscript 0. Then,

NMF2α $\Gamma \vdash_{\text{NMF2}\alpha} \mathcal{A}$ iff there is an NMF2α derivation of $\mathcal{A}_0$ from the members of Γ_0.

Here are a couple of examples, the first related to one from the previous appendix, but exercising the presupposition rule, the second a simple derivation of the necessary indiscerniblity of identicals.

$\exists w \Diamond \neg E w \vdash_{\text{NMF2}\alpha} \neg \forall x \Box H x$

1	$\exists w \Diamond \neg E w_0$	P
2	$\Diamond \neg E a_0$	A $(g, 1\ \exists E)$
3	$E a_0$	
4	0.1	A $(g, 2\ \Diamond E)$
5	$\neg E a_1$	
6	$\forall x \Box H x_0$	A $(g, \neg I)$
7	$\Box H a_0$	6,3 $\forall E$
8	$H a_1$	7,4 $\Box E$
9	$E a_1$	8 $P_2 I$
10	$\neg E a_1$	5 R
11	$\neg \forall x \Box H x_0$	6-10 $\neg I$
12	$\neg \forall x \Box H x_0$	2,4-11 $\Diamond E$
13	$\neg \forall x \Box H x_0$	1,2-12 $\exists E$

$\vdash_{\text{NMF2}\alpha} a = b \rightarrow \Box \forall \text{x}(I \text{x} a \leftrightarrow I \text{x} b)$

1	$(a = b)_0$	A $(g \rightarrow I)$
2	0.1	A $(g\ \Box I)$
3	$E \text{R}_1$	A $(g\ \forall I)$
4	$I \text{R} a_1$	A $(g \leftrightarrow I)$
5	$I \text{R} b_1$	4,1 $=E$
6	$I \text{R} b_1$	A $(g \leftrightarrow I)$
7	$I \text{R} a_1$	6,1 $=E$
8	$(I \text{R} a \leftrightarrow I \text{R} b)_1$	4-5,6-7 $\leftrightarrow I$
9	$\forall \text{x}(I \text{x} a \leftrightarrow I \text{x} b)_1$	3-8 $\forall I$
10	$\Box \forall \text{x}(I \text{x} a \leftrightarrow I \text{x} b)_0$	2-9 $\Box I$
11	$[a = b \rightarrow \Box \forall \text{x}(I \text{x} a \leftrightarrow I \text{x} b)]_0$	1-10 $\rightarrow I$

Appendix D

Adding Existence Presuppositions

We have noted the options that (1) Hb is either true or false where Bill does not exist; (2) Hb is false where Bill does not exist; and (3) Hb is neither true nor false where Bill does not exist. So far, we have seen options for (1), and then (2). In this appendix, I extend the language to include individual constants and variables of two sorts, such that if terms are restricted to one sort or the other, the result is option (2) or (3). The language with mixed terms results in a combination of the two options. The combined option is utilized in the main text, with its distinction between proper and ordinary things. For soundness and completeness see [TR6.1, 6.2].

D.1   Language / Semantic Notions

LMIFnα For a language $\mathcal{L}_{\mu\nu}^{S}$, the *vocabulary* consists of sets of individual variables both x_η, and x_η, relation variables $\mathrm{x}_\eta^{\mathrm{n}}$, individual constants both a_η and a_η, relation constants $\mathrm{A}_\eta^{\mathrm{n}}$, predicate symbols, I^{n+1}, E, $=$, R_η^n; and operators $\neg$, $\bigwedge$, $\forall$, $\square$. The number of "places" in a relation variable, relation constant or predicate symbol is indicated by superscript, where '$=$' is always two-place, and 'E' one-place. Any variable or constant is a *term*. If $\mathcal{Q}^n t_1 \ldots t_n$ is an n-place predicate symbol followed by n terms, it is a *formula*, but $I^{n+1}\mathrm{R}^{\mathrm{n}}t_1 \ldots t_n$ is well-formed iff R^{n} is a n-place relation variable or constant and $t_1 \ldots t_n$ are n individual terms.

If $\mathcal{P}$ is a formula, Π is a set of formulas with $|\Pi| < \mu$, Ξ is a set of variables with $|\Xi| < \nu$, then $\neg\mathcal{P}$, $\square\mathcal{P}$, $\bigwedge\Pi$, and $\forall\Xi\mathcal{P}$ are *formulas*. Variables are bound and free in the usual way. $\mathcal{P}$ is a *sentence* iff it is a formula with no free variables. Allow the usual abbreviations.

IMIFnα An *interpretation* is $\langle W, O, P, R, D, A, V \rangle$ where W is a set of worlds; O and P are sets of objects (named to suggest ordinary and proper things); R is a sequence of sets of relations, such that $\textsc{r} \in \text{Rn}$ is a (partial) function from W to subsets of U^n; D is a function from W to sets of objects and relations from O, P and the union of R; and A is a subset of W^2. Then $D_O(w) = D(w) \cap O$, $D_P(w) = D(w) \cap P$, $D_R(w) = D(w) \cap \bigcup R$ and $D_{Rn}(w) = D(w) \cap Rn$. Require that the domain of any $\textsc{r} \in \text{Rn}$ is $\{w \mid \textsc{r} \in D(w)\}$, and for any world in its domain, $\textsc{r}^{\textsc{n}}(w) \subseteq D_U(w)^n$. V is a function such that for any constant a, $V[a] \in O$; for any constant $\mathcal{a}$, $V[\mathcal{a}] \in P$; for any relation constant $\textsc{a}^{\textsc{n}}$, $V[\textsc{a}^{\textsc{n}}] \in \text{Rn}$; and for any predicate symbol, $\mathcal{R}^n$, and $w \in W$, $V[w, \mathcal{R}^n] \subseteq D(w)^n$. Require that $V[w, E] = D(w)$; $V[w, =] = \{\langle u, u \rangle \mid u \in D(w)\}$; and $V[w, I^{n+1}] = \{\langle \textsc{r}^{\textsc{n}}, u_1 \dots u_n \rangle \mid \langle u_1 \dots u_n \rangle \in \textsc{r}^{\textsc{n}}(w)\}$. In addition, where α is empty or some combination of the following,

ρ	for all x, xAx	reflexivity
σ	for all x, y, if xAy then yAx	symmetry
τ	for all x, y, z, if xAy and yAz then xAz	transitivity
υ	for all x, y, xAy	universality

$\langle W, O, P, R, D, A, V \rangle$ is a MIFnα interpretation when A satisfies constraints from α.

A variable designation assignment d assigns each variable x a member of O, each $\mathcal{x}$ a member of P, and each $\textsc{x}^{\textsc{n}}$ a member of Rn. Where Ξ is a set of variables and f is a function that takes any $x \in \Xi$ to a member of O, any $\mathcal{x} \in \Xi$ to a member of P, and any $\textsc{x}^{\textsc{n}} \in \Xi$ to a member of Rn, $d(\Xi|f)$ is like d except that the assignment to any $x \in \Xi$ is $f(x)$. Then set $\langle u_1 \dots u_n \rangle \in \overline{V}[w, \mathcal{Q}^n]$ iff $\langle u_1 \dots u_n \rangle \notin V[w, \mathcal{Q}^n]$, and for any $u_i \in P$, $u_i \in D_P(w)$.

TNIF For assignments to formulas,

> (R) $V_d[w, \mathcal{Q}^n t_1 \dots t_n] = 1$ iff $\langle V_d[t_1] \dots V_d[t_n] \rangle \in V[w, \mathcal{Q}^n]$;
> $V_d[w, \mathcal{Q}^n t_1 \dots t_n] = 0$ iff $\langle V_d[t_1] \dots V_d[t_n] \rangle \in \overline{V}[w, \mathcal{Q}^n]$.
>
> ($\neg$) $V_d[w, \neg \mathcal{A}] = 1$ iff $V_d[w, \mathcal{A}] = 0$;
> $V_d[w, \neg \mathcal{A}] = 0$ iff $V_d[w, \mathcal{A}] = 1$.
>
> ($\wedge$) $V_d[w, \bigwedge \Pi] = 1$ if $V_d[w, \mathcal{A}] = 1$ for each $\mathcal{A} \in \Pi$;
> $V_d[w, \bigwedge \Pi] = 0$ if $V_d[w, \mathcal{A}] = 0$ for some $\mathcal{A} \in \Pi$.
>
> ($\forall$) $V_d[w, \forall \Xi \mathcal{A}] = 1$ iff for any f from Ξ to $D(w)$, $V_{d(\Xi|f)}[w, \mathcal{A}] = 1$;
> $V_d[w, \forall \Xi \mathcal{A}] = 0$ iff for some f from Ξ to $D(w)$, $V_{d(\Xi|f)}[w, \mathcal{A}] = 0$.

($\square$) $V_d[w, \square \mathcal{A}] = 1$ iff all $x \in W$ such that wAx have $V_d[x, \mathcal{A}] = 1$;

$\qquad$ $V_d[w, \square \mathcal{A}] = 0$ iff some $x \in W$ such that wAx has $V_d[x, \mathcal{A}] = 0$.

$V[w, \mathcal{A}] = 1$ iff for any d, $V_d[w, \mathcal{A}] = 1$. And $V[w, \Gamma] = 1$ iff for each $\mathcal{A} \in \Gamma$, $V[w, \mathcal{A}] = 1$. Then, where the members of Γ and $\mathcal{A}$ are sentences of $\mathcal{L}^S_{\mu\nu}$,

VMIFnα $\Gamma \vDash_{\overline{\text{MIFn}\alpha}} \mathcal{A}$ iff there is no MFnα interpretation $\langle W, O, P, R, D, A, V \rangle$ for
$\qquad$ $\mathcal{L}^S_{\mu\nu}$ and $w \in W$ such that $V[w, \Gamma] = 1$ and $V[w, \mathcal{A}] \neq 1$.

If P is empty and the language is restricted to constants a and ʀ and variables x and x, the result is as before. This is option (2). If O is empty and the language is restricted to constants a and variables x, then both $V_d[w, \mathcal{Q}t_1 \dots t_n] \neq 1$ and $V_d[w, \mathcal{Q}t_1 \dots t_n] \neq 0$ in any case where some of $V_d[t_1] \dots V_d[t_n]$ are not in $D_p(w)$; the classical rules for truth and falsity apply to expressions that have values (as in the "strong" Kleene tables). This is an implementation of option (3). The full system combines the two.

D.2 Natural Derivations: *NMFnα*

Introduce expressions of the sort $\mathcal{A}$ and $\overline{\mathcal{A}}$. Intuitively $\overline{\mathcal{A}}$ indicates that $\mathcal{A}$ is *not false*. Let $\backslash\mathcal{A}\backslash$ and $/\mathcal{A}/$ represent either $\mathcal{A}$ or $\overline{\mathcal{A}}$ where what is represented is constant in a given context, but $\backslash\mathcal{A}\backslash$ and $/\mathcal{A}/$ are opposite. And similarly for $\!/\!\!/\mathcal{A}/\!\!/$ and $\backslash\!\backslash\mathcal{A}\backslash\!\backslash$, though there need be no fixed relation between overlines on $\backslash\mathcal{A}\backslash$ and $\backslash\!\backslash\mathcal{A}\backslash\!\backslash$. Rules are a natural development of ones we have already seen.

$\vee\mathbf{I}$ $\begin{vmatrix} /\mathcal{P}/_s \\ \\ /\mathcal{P}\vee\mathcal{Q}/_s \end{vmatrix}$ $\qquad$ $\vee\mathbf{I}$ $\begin{vmatrix} /\mathcal{P}/_s \\ \\ /\mathcal{Q}\vee\mathcal{P}/_s \end{vmatrix}$ $\qquad$ $\vee\mathbf{E}$ $\begin{vmatrix} /\mathcal{P}\vee\mathcal{Q}/_s \\ \quad| /\mathcal{P}/_s \\ \quad| \\ \quad| /\!/\mathcal{R}/\!/_t \\ \quad| /\mathcal{Q}/_s \\ \quad| \\ \quad| /\!/\mathcal{R}/\!/_t \\ /\!/\mathcal{R}/\!/_t \end{vmatrix}$

$\rightarrow\mathbf{I}$ $\begin{vmatrix} | /\mathcal{P}/_s \\ | \\ | \backslash\mathcal{Q}\backslash_s \\ \backslash\mathcal{P}\rightarrow\mathcal{Q}\backslash_s \end{vmatrix}$ $\qquad$ $\rightarrow\mathbf{E}$ $\begin{vmatrix} \backslash\mathcal{P}\rightarrow\mathcal{Q}\backslash_s \\ /\mathcal{P}/_s \\ \\ \backslash\mathcal{Q}\backslash_s \end{vmatrix}$

$\leftrightarrow\mathbf{I}$ $\begin{vmatrix} | /\mathcal{P}/_s \\ | \\ | \backslash\mathcal{Q}\backslash_s \\ | /\mathcal{Q}/_s \\ | \\ | \backslash\mathcal{P}\backslash_s \\ \backslash\mathcal{P}\leftrightarrow\mathcal{Q}\backslash_s \end{vmatrix}$ $\qquad$ $\leftrightarrow\mathbf{E}$ $\begin{vmatrix} \backslash\mathcal{P}\leftrightarrow\mathcal{Q}\backslash_s \\ /\mathcal{P}/_s \\ \\ \backslash\mathcal{Q}\backslash_s \end{vmatrix}$ $\qquad$ $\leftrightarrow\mathbf{E}$ $\begin{vmatrix} \backslash\mathcal{P}\leftrightarrow\mathcal{Q}\backslash_s \\ /\mathcal{Q}/_s \\ \\ \backslash\mathcal{P}\backslash_s \end{vmatrix}$

$\forall\mathbf{I}$ $\begin{vmatrix} | E\,a_s \\ | \\ | /\mathcal{P}_a^x/_s \\ /\forall x\mathcal{P}/_s \end{vmatrix}$ $\quad$ $\forall\mathbf{E}$ $\begin{vmatrix} /\forall x\mathcal{P}/_s \\ E\,a_s \\ \\ /\mathcal{P}_a^x/_s \end{vmatrix}$ $\quad$ $\exists\mathbf{I}$ $\begin{vmatrix} /\mathcal{P}_a^x/_s \\ E\,a_s \\ \\ /\exists x\mathcal{P}/_s \end{vmatrix}$ $\quad$ $\exists\mathbf{E}$ $\begin{vmatrix} /\exists x\mathcal{P}/_s \\ \quad| /\mathcal{P}_a^x/_s \\ \quad| E\,a_s \\ \quad| \\ \quad| /\!/\mathcal{Q}/\!/_u \\ /\!/\mathcal{Q}/\!/_u \end{vmatrix}$

where a does not appear in any undischarged premise or assumption or in $\mathcal{P}$

$\qquad\qquad$ – x and a of the same type –

where a does not appear in any undischarged premise or assumption, in $\mathcal{P}$ or in $\mathcal{Q}$

$\Box\mathbf{I}$ $\begin{vmatrix} | s.t \\ | \\ | /\mathcal{P}/_t \\ /\Box\mathcal{P}/_s \end{vmatrix}$ $\quad$ $\Box\mathbf{E}$ $\begin{vmatrix} /\Box\mathcal{P}/_s \\ s.t \\ \\ /\mathcal{P}/_t \end{vmatrix}$ $\quad$ $\Diamond\mathbf{I}$ $\begin{vmatrix} /\mathcal{P}/_t \\ s.t \\ \\ /\Diamond\mathcal{P}/_s \end{vmatrix}$ $\quad$ $\Diamond\mathbf{E}$ $\begin{vmatrix} /\Diamond\mathcal{P}/_s \\ \quad| s.t \\ \quad| /\mathcal{P}/_t \\ \quad| \\ \quad| /\!/\mathcal{Q}/\!/_u \\ /\!/\mathcal{Q}/\!/_u \end{vmatrix}$

where t does not appear in any undischarged premise or assumption

where t does not appear in any undischarged premise or assumption and is not u

$\mathbf{AM}\rho$ $\begin{vmatrix} \\ \\ s.s \end{vmatrix}$ $\qquad$ $\mathbf{AM}\sigma$ $\begin{vmatrix} s.t \\ \\ t.s \end{vmatrix}$ $\qquad$ $\mathbf{AM}\tau$ $\begin{vmatrix} s.t \\ t.u \\ \\ s.u \end{vmatrix}$ $\qquad$ $\mathbf{AM}\upsilon$ $\begin{vmatrix} \\ \\ s.t \end{vmatrix}$

For an atomic sentence $\mathcal{Q}\,a_1 \ldots a_n$ let $P_2[\mathcal{Q}\,a_1 \ldots a_n]$ be as before. $P_3[\mathcal{Q}\,a_1 \ldots a_n]_s = (\top \wedge E\,a_c \wedge \ldots E\,a_e)_s$ for any $a_c \ldots a_e$ in $a_1 \ldots a_n$.

$$
\mathbf{D}\ \left|\ \begin{array}{l} \mathcal{P}_s \\[1ex] \overline{\mathcal{P}}_s \end{array} \right. \qquad
\mathbf{rU}\ \left|\ \begin{array}{l} \overline{(\mathcal{Q}\,a_1 \ldots a_n)}_s \\ P_3[\mathcal{Q}\,a_1 \ldots a_n]_s \\[1ex] (\mathcal{Q}\,a_1 \ldots a_n)_s \end{array} \right. \qquad
\mathbf{P_2I}\ \left|\ \begin{array}{l} (\mathcal{Q}\,a_1 \ldots a_n)_s \\[1ex] P_2[\mathcal{Q}\,a_1 \ldots a_n]_s \end{array} \right. \qquad
\mathbf{P_3I}\ \left|\ \begin{array}{l} (\neg\mathcal{Q}\,a_1 \ldots a_n)_s \\[1ex] P_3[\mathcal{Q}\,a_1 \ldots a_n]_s \end{array} \right.
$$

$$
\mathbf{=I}\ \left|\ \begin{array}{l} E\,a_s \\[1ex] (a = a)_s \end{array} \right. \qquad
\mathbf{=E}\ \left|\ \begin{array}{l} (a = c)_s \\ \|\mathcal{P}\|_t \\[1ex] \|\mathcal{P}^{a}/_{c}\|_t \end{array} \right. \quad \left|\ \begin{array}{l} (c = a)_s \\ \|\mathcal{P}\|_t \\[1ex] \|\mathcal{P}^{a}/_{c}\|_t \end{array} \right. \qquad
\mathbf{Iso}\ \left|\ \begin{array}{l} E\,a_s \\ /\mathcal{P}/_s \\ \backslash\neg\mathcal{P}\backslash_t \\[1ex] \overline{\neg E\,a}_t \end{array} \right.
$$

one or more instances of a replaced by c

(D) is for truth *down* and (rU) a restricted truth *up*. In case the language has no constants of the sort a, $P_3[\mathcal{Q}\,a_1 \ldots a_n]$ is trivial, so that (rU) and, by a simple induction, the up rule applies to any formula. Then things work as in NMF2α so that the logic is entirely classical. The rules for NMFnα take appropriate AM rules and exclude (Iso). The latter plays a role in the theory for the main text and, intuitively, requires that $D_p(w)$ be disjoint from $D_p(x)$ when $w \not\preceq x$ and $x \not\preceq w$. Where the members of Γ and $\mathcal{A}$ are sentences without overlines or subscripts, let Γ_0 be the members of Γ, each with subscript 0. Then,

NMFnα $\quad \Gamma \vdash_{\text{NMFn}\alpha} \mathcal{A}$ iff there is an NMFnα derivation of $\mathcal{A}_0$ from the members of Γ_0.

Here are a couple of examples, the first exercising just sentential rules with overlines, the second again like one we have seen before.

$\neg\Box\neg P \vdash_{\text{NFn}\alpha} \Diamond P$

1	$\neg\Box\neg P_0$	P
2	$\overline{\neg\Diamond P_0}$	A (c ¬E)
3	0.1	A (g □I)
4	P_1	A (c ¬I)
5	$\Diamond P_0$	4,3 ◇I
6	$\overline{\neg\Diamond P_0}$	2 R
7	$\overline{\neg P_1}$	4-6 ¬I
8	$\overline{\Box\neg P_0}$	3-7 □I
9	$\neg\Box\neg P_0$	1 R
10	$\Diamond P_0$	2-9 ¬E

$\exists w \Diamond \neg Ew \vdash_{\text{NFn}\alpha} \neg\forall x \Box Hx$

1	$\exists w \Diamond \neg Ew_0$	P
2	$\Diamond\neg Ea_0$	A (g, 1 ∃E)
3	Ea_0	
4	0.1	A (g, 2 ◇E)
5	$\neg Ea_1$	
6	$\forall x \Box Hx_0$	A (g, ¬I)
7	$\overline{\Box Ha_0}$	6,3 ∀E
8	$\overline{Ha_1}$	7,4 □E
9	Ha_1	8 rU
10	Ea_1	9 P_2I
11	$\overline{\neg Ea_1}$	5 D
12	$\neg\forall x \Box Hx_0$	6-11 ¬I
13	$\neg\forall x \Box Hx_0$	2,4-12 ◇E
14	$\neg\forall x \Box Hx_0$	1,2-13 ∃E

The first example illustrates one part of a standard equivalence between $\neg\Box\neg\mathcal{P}$ and $\Diamond\mathcal{P}$. The derivation illustrates especially negation rules: Where a sentence may be neither true nor false, we reject an assumption when $\mathcal{P}$ is demonstrably true and not true. Step (9) in the second is legitimate insofar as $P_3[Ha]_1$ is trivial. The argument would not go through to show $\exists w \Diamond \neg Ew \vdash_{\text{NFn}\alpha} \neg\forall x \Box Hx$ insofar as that step is blocked. The argument would not go through in NF1α insofar as P_2I is required at (10).

Bibliography

Adams, Robert. "Actualism and Thisness." *Synthese* 49 (1981): 3–41.

Armstrong, David. *A Combinatorial Theory of Possibility*. Cambridge: Cambridge University Press, 1989a.

———. *Universals, An Opinionated Introduction*. Boulder: Westview Press, 1989b.

———. *A World of States of Affairs*. Cambridge: Cambridge University Press, 1997.

Baker, Lynne Rudder. *Persons and Bodies: A Constitution View*. Cambridge: Cambridge University Press, 2000.

Balaguer, Mark. *Platonism and Anti-Platonism in Mathematics*. Oxford: Oxford University Press, 1998.

Benacerraf, and Putnam, editors. *Philosophy of Mathematics: Selected Readings*. Cambridge: Cambridge University Press, 1983, second edition.

Benacerraf, Paul. "What Numbers Could Not Be." *Philosophical Review* 74 (1965): 47–73. Reprint in many places including Benacerraf and Putnam, *Philosophy of Mathematics*, 272-294.

———. "Mathematical Truth." *Journal of Philosophy* 70 (1973): 661–680. Reprint in many places including Benacerraf and Putnam, *Philosophy of Mathematicss*, 403-420.

Bennett, Karen. "Spatio-Temporal Coincidence and the Grounding Problem." *Philosophical Studies* 118 (2004): 339–371.

Boolos, Burgess, and Jeffrey. *Computability and Logic*. Cambridge: Cambridge University Press, 2002, fourth edition.

BIBLIOGRAPHY

Boolos, George. "To Be Is To Be a Value of a Variable (or to Be Some Values of Some Variables)." *Journal of Philosophy* 81 (1984): 430–450. Reprint in Boolos, *Logic, Logic and Logic*, 54-72.

———. "Nominalist Platonism." *Philosophical Review* 94 (1985a): 331–344.

———. "Reading the *Begriffsschrift*." *Mind* 94 (1985b): 331–334. Reprint in *Logic, Logic and Logic*, 155-170.

———. "Iteration Again." *Philosophical Topics* 42 (1989): 5–21. Reprint in Boolos, *Logic, Logic and Logic*, 88-104.

———. *Logic, Logic and Logic*. Cambridge: Harvard University Press, 1998.

———. "Must We Believe in Set Theory?" In *Between Logic and Intuition: Essays in Honor of Charles Parsons*, edited by Sher, and Tieszen, Cambridge: Cambridge University Press, 2000, 257–268. Reprint in Boolos, *Logic, Logic and Logic*, 120-132.

Brock, Stuart. "Modal Fictionalism: A Response to Rosen." *Mind* 102 (1993): 147–150.

Burgess, John, and Gideon Rosen. *A Subject With No Object*. Oxford: Oxford University Press, 1997.

Burke, Michael. "Copper Statues and Pieces of Copper: A Challenge to the Standard Account." *Analysis* 52 (1992): 12–117.

———. "Preserving the Principle of One Object to a Place: A Novel Account of the Relations Among Objects, Sorts, Sortals, and Persistence Conditions." *Philosophy and Phenomenological Research* 54 (1994): 591–624. Reprint in Rea, *Material Constitution*, 236-269.

Callender, Craig. "Humean Supervenience and Rotating Homogeneous Matter." *Mind* 110 (2001): 25–44.

Chisholm, Roderick. "Identity Through Possible Worlds: Some Questions." *Nous* 1 (1967): 1–8. Reprint in Loux, *The Possible and the Actual*, 80-87.

Clark, Romane. "Concerning the Logic of Predicate Modifiers." *Noûs* 4 (1970): 311–335.

Davidson, Donald. "The Logical Form of Action Sentences." In *The Logic of Decision and Action*, edited by Rescher, Pittsburgh: University of Pittsburgh Press, 1967, 81–120. Updated reprint in Davidson, *Essays on Actions and Events*, 105-122.

———. "The Individuation of Events." In *Essays in Honor of Carl G. Hempel*, edited by Rescher, Dordrecht: D. Reidel, 1969, 216–234. Reprint in Davidson, *Essays on Actions and Events*, 265-283.

———. *Essays on Actions and Events*. Oxford: Clarendon Press, 1980.

Davidson, Matthew. "Transworld Identity, Singular Propositions and Picture-Thinking." In *On Sense and Direct Reference: Readings in the Philosophy of Language*, edited by Davidson, Boston: McGraw-Hill, 2007, 559–568.

Della Rocca, Michael. "Essentialists and Essentialism." *Journal of Philosophy* 93 (1996): 186–202.

Denkel, Arda. *Object and Property*. Cambridge: Cambridge University Press, 1996.

Devitt, Michael. *Realism and Truth*. Princeton: Princeton University Press, 1997, second edition.

Dickmann, M. "Larger Infinitary Languages." In *Model-Theoretic Logics*, edited by Barwise, and Feferman, New York: Springer-Verlag, 1985, 317–363.

Divers, John. "Modal Fictionalism Cannot Deliver Possible Worlds Semantics." *Analysis* 55 (1995): 81–88.

———. "The Analysis of Possibility and the Possibility of Analysis." *Proceedings of the Aristotelian Society* 97 (1997): 141–160.

Donnelly, Maureen. "UsingMereological Principles to Support Metaphysics." *The Philosophical Quarterly* 61 (2011): 225–246.

Drake, and Singh. *Intermediate Set Theory*. New York: John Wiley and Sons, 1996.

Enderton, Herbert. *A Mathematical Introduction to Logic*. San Diego: Academic Press, 1972.

Field, Hartry. *Realism, Mathematics and Modality*. Oxford: Blackwell, 1989.

Fine, Kit. "Things and Their Parts." *Midwest Studies in Philosophy* 23 (1999): 61–74.

———. "The Problem of Possibilia." In *The Oxford Handbook of Metaphysics*, Oxford: Oxford University Press, 2003, 161–179.

———. "Towards a Theory of Part." *The Journal of Philosophy* 107 (2010): 559–589.

Garson, James. "Quantification in Modal Logic." In *Handbook of Philosophical Logic, Volume 3*, Dordrecht: Kluwer Academic Publishers, 2001, 267–323. Second edition.

Geach, Peter. *Reference and Generality*. Ithaca: Cornell University Press, 1980.

Gellner, Ernest. "The Last Pragmatist, or the Behaviourist Platonist." In *Spectacles and Predicaments: Essays in Social Theory*, edited by Gellner, Cambridge: Cambridge University Press, 1979, 199–208.

Gettier, Edmund. "Is Justified True Belief Knowledge?" *Analysis* 23 (1963): 121–123. Reprint in many places.

Gibbard, Allan. "Contingent Identity." *Journal of Philosophical Logic* 4 (1975): 187–221.

Gödel, Kurt. "What Is Cantor's Continuum Problem?" *American Mathematical Monthly* 54 (1947): 515–525. Revised version in Benacerraf and Putnam, *Philosophy of Mathematics*, 470-485.

Goldman, Alvin. "A Causal Theory of Knowing." *Journal of Philosophy* 64 (1967): 357–372.

Grim, Patrick. *The Incomplete Universe: Totality, Knowledge, and Truth*. Cambridge, MA: MIT Press, 1991.

Hale, Bob. *Abstract Objects*. Oxford: Basil Blackwell, 1987.

———. "Modal Fictionalism: A Simple Dilemma." *Analysis* 55 (1995): 63–67.

Hawley, Katherine. "Persistence and Non-Supervenient Relations." *Mind* 108 (1999): 53–67.

Heller, Mark. *The Ontology of Physical Objects:Four-Dimensional Hunks of Matter*. Cambridge: Cambridge University Press, 1990.

Hellman, Geoffrey. *Mathematics Without Numbers: Towards a Modal-Structural Interpretation*. Oxford: Oxford University Press, 1989.

———. "Structuralism Without Structures." *Philosophia Mathematica* 3 (1996): 100–123.

———. "Structuralism." In *The Oxford Handbook of Philosophy of Mathematics and Logic*, edited by Stewart Shapiro, Oxford: Oxford University Press, 2005, 536–562.

Hintikka, Jaakko. "The Modes of Modality." *Acta Philosophica Fennica* 16 (1963): 65–79.

Hoffman, Joshua, and Gary Rosenkrantz. *Substance Among Other Categories*. Cambridge: Cambridge University Press, 1994.

Humberstone, I. L. "From Worlds to Possibilities." *Journal of Philosophical Logic* 10 (1981): 313–339.

Jackson, Frank. "A Causal Theory of Counterfactuals." *Australasian Journal of Philosophy* 55 (1977): 3–21.

Jeffrey, Richard. *The Logic of Decision*. New York: McGraw-Hill, 1965.

Johnston, Mark. "Parts and Principles: False Axioms in Mereology." *Philosophical Topics* 30 (2002): 129–166.

———. "Hylomorphism." *The Journal of Philosoph* 103 (2006): 6552–698.

Jubien, Michael. "Problems With Possible Worlds." In *Philosophical Analysis*, edited by Austin, Boston: Kluwer Academic Publishers, 1988, 299–322.

———. "On Properties and Property Theory." In *Properties, Types and Meaning*, edited by Chierchia, Partee, and Turner, Boston: Kluwer Academic Publishers, 1989a, 159–175.

———. "Straight Talk About Sets." *Philosophical Topics* 17 (1989b): 91–107.

———. *Ontology, Modality, and the Fallacy of Reference*. Cambridge: Cambridge University Press, 1993.

———. "Actualism and Iterated Modalities." *Philosophical Studies* 84 (1996): 109–125.

———. *Possibility*. Oxford: Oxford University Press, 2009.

Kaplan, David. "Transworld Heir Lines." In *The Possible and the Actual*, edited by Loux, Ithaca: Cornell University Press, 1979, 88–109.

Kim, Jaegwon. "Causation, Nomic Subsumption, and the Concept of Event." *Journal of Philosophy* 70 (1973): 217–236. Reprint in Kim, *Supervenience and Mind*, 3-21.

———. "Events as Property Exemplifications." In *Action Theory*, edited by Brand, and Walton, Dordrecht: D. Reidel Publishing Co., 1976, 159–177. Reprint in Kim, *Supervenience and Mind*, 33-52.

———. "Concepts of Supervenience." *Philosophy and Phenomenological Research* 45 (1984): 153–176. Reprint in Kim, *Supervenience and Mind*, 53-78.

———. "Supervenience for Multiple Domains." *Philosophical Topics* 16 (1988): 129–150. Reprint in Kim, *Supervenience and Mind*, 109-130.

———. "Supervenience as a Philosophical Concept." *Metaphilosophy* 21 (1990): 1–27. Reprint in Kim, *Supervenience and Mind*, 131-160.

———. *Supervenience and Mind*. New York: Cambridge University Press, 1993.

———. *Mind in a Physical World*. Cambridge, Mass.: MIT Press, 1998.

Kim, Seahwa. "Modal Fictionalism and Analysis." In *Fictionalism in Metaphysics*, edited by Mark Kalderon, Oxford: Oxford University Press, 2005, 116–133.

Koslicki, Kathrin. *The Structure of Objects*. Oxford: Oxford University Press, 2008.

Kripke, Saul. "Semantical Analysis of Modal Logic I: Normal Propositional Calculi." *Zeitschrift für Mathematische Logik und Grundlagen der Mathematik* 8 (1963a): 67–96.

———. "Semantical Considerations on Modal Logic." *Acta Philosophica Fennica* 16 (1963b): 83–94.

———. *Naming and Necessity*. Cambridge, MA: Harvard University Press, 1980.

Lewis, David. "Counterparts of Persons and Their Bodies." *Journal of Philosophy* 68 (1971): 203–211. Reprint in Lewis, *Philosophical Papers, Vol I*, 47-54.

———. *Counterfactuals*. Oxford: Basil Blackwell, 1973.

———. "Truth in Fiction." *American Philosophical Quarterly* 15 (1978): 37–46. Reprint in Lewis, *Philosophical Papers, Vol I*, 261-280.

———. "A Subjectivist's Guide to Objective Chance." In *Studies in Inductive Logic and Probability, Volume II*, edited by Jeffrey, Berkeley: University of California Press, 1980, 263–294. Reprint in Lewis, *Philosophical Papers, Vol II*, 83-113.

———. *Philosophical Papers, Volume I.* Oxford: Oxford University Press, 1983.

———. "Putnam's Paradox." *The Australasian Journal of Philosophy* 62 (1984): 231–236. Reprint in Lewis, *Papers in Metaphysics and Epistemology*, 56-77.

———. "Introduction." In *Philosophical Papers, Volume II*, Oxford: Oxford University Press, 1986a, ix–xvii.

———. *On The Plurality of Worlds.* New York: Basil Blackwell, 1986b.

———. *Philosophical Papers, Volume II.* Oxford: Oxford University Press, 1986c.

———. "Postscripts to 'A Subjectivist's Guide to Objective Chance'." In *Philosophical Papers, Volume II*, Oxford: Oxford University Press, 1986d, 114–132.

———. *Parts of Classes.* Oxford: Basil Blackwell, 1991.

———. "Many, But Almost One." In *Ontology, Causality, and Mind: Essays on the Philosophy of D. M. Armstrong*, edited by Cambell, Bacon, and Reinhardt, Cambridge: Cambridge University Press, 1993, 23–37. Reprint in Lewis, *Papers in Metaphysics and Epistemology*, 164-182.

———. "Humean Supervenience Debugged." *Mind* 103 (1994): 473–490. Reprint in Lewis, *Papers in Metaphysics and Epistemology*, 224-247.

———. *Papers in Metaphysics and Epistemology.* Cambridge: Cambridge University Press, 1999a.

———. "Zimmerman and the Spinning Sphere." *Australasian Journal of Philosophy* 77 (1999b): 209–212.

Linsky, and Zalta. "Naturalized Platonism Versus Platonized Naturalism." *The Journal of Philosophy* 92 (1995): 525–555.

Loux, editor. *The Possible and the Actual: Readings in the Metaphysics of Modality.* Ithaca: Cornell University Press, 1979.

Mackie, J. L. *Ethics: Inventing Right and Wrong.* New York: Penguin Books, 1977.

Maddy, Penelope. *Realism in Mathematics.* Oxford: Oxford University Press, 1990.

Marcus, Ruth. "Modalities and Intensional Languages." *Synthese* 13 (1961): 303–322.

McGee, Vann, and Brian McLaughlin. "The Lessons of the Many." *Philosophical Topics* 28 (2000): 129–151.

McGinn, Colin. "Modal Reality." In *Reduction, Time and Reality*, edited by Healey, Cambridge: Cambridge University Press, 1981, 143–187.

McMichael, Alan. "A Problem for Actualism About Possible Worlds." *Philosophical Review* 92 (1983): 49–66.

Merricks, Trenton. *Objects and Persons*. Oxford: Oxford University Press, 2001.

———. "Review of *The Structure of Objects*." *The Journal of Philosophy* 106 (2009): 301–307.

Mondadori, and Morton. "Modal Realism: The Poisoned Pawn." *Philosophical Review* 85 (1976): 3–20. Reprint in Loux, *The Possible and the Actual*, 235-252.

Montague, Richard. "Semantical Closure and Non-Finite Axiomatizability I." In *Infinitistic Methods: Proceedings of the Symposium on Foundations of Mathematics*, New York: Pergamon Press, 1961, 45–69.

Nadel, M. "$\mathcal{L}_{\omega_1\omega}$ and Admissible Fragments." In *Model-Theoretic Logics*, edited by Barwise, and Feferman, New York: Springer-Verlag, 1985, 271–316.

Nolan, Daniel. "Three Problems for "Strong" Modal Fictionalism." *Philosophical Studies* 87 (1997): 259–275.

———. "Modal Fictionalism." In *The Stanford Encyclopedia of Philosophy*, edited by Edward Zalta, Summer 2002. URL http://plato.stanford.edu/archives/sum2002/entries/fictionalism-modal/.

Noonan, Harold. "The Closest Continuer Theory of Identity." *Inquiry* 28 (1985): 195–229.

Oliver, and Smiley. *Plural Logic*. Oxford: Oxford University Press, 2013.

Parsons, Charles. "What is the Iterative Conception of Set?" In *Proceedings of the 5th International Congress of Logic, Methodology and Philosophy of Science 1975, Part I: Logic, Foundations of Mathematics, and Computability Theory*, edited by Butts, and Hintikka, Dordrecht: D. Reidel, 1977, 335–367. Reprint in Benacerraf and Putnam, *Philosophy of Mathematics*, 503-529.

Parsons, Terence. *Nonexistent Objects*. New Haven: Yale University Press, 1980.

Plantinga, Alvin. *God, Freedom, and Evil*. Grand Rapids: Eerdmans Publishing Company, 1974a.

———. *The Nature of Necessity*. Oxford: Oxford University Press, 1974b.

———. "Actualism and Possible Worlds." *Theoria* 42 (1976): 139–160. Reprint in Loux, *The Possible and the Actual*, 253-273.

Potter, Michael. *Set Theory and its Philosophy*. Oxford: Oxford University Press, 2004.

Priest, Graham. *An Introduction to Non-Classical Logic: From If to Is*. Cambridge: Cambridge University Press, 2008, second edition.

Putnam, Hilary. "Models and Reality." *Journal of Symbolic Logic* 45 (1980): 464–482. Reprint in Putnam, *Realism and Reason*, 1-25.

———. *Reason, Truth and History*. Cambridge: Cambridge University Pres, 1981.

———. *Realism and Reason: Philosophical Papers, vol. 3*. Cambridge: Cambridge University Press, 1983.

———. *The Many Faces of Realism*. LaSalle: Open Court, 1987.

Quine, Willard Van Orman. "On What There Is." *Review of Metaphysics* 2 (1948): 21–38. Reprint in many places including Quine, *From a Logical Point of View*, 1-19.

———. "Three Grades of Modal Involvement." In *Proceedings of the XIth International Congress of Philosophy, Brussels 1953, Volume 14*, Amsterdam: North Holland Publishing Company, 1953. Reprint in Quine, *The Ways of Paradox*, 158-176.

———. *The Ways of Paradox and Other Essays*. Cambridge, MA: Harvard University Press, 1976, second edition.

———. *From a Logical Point of View*. Cambridge: Harvard University Press, 1980a, second edition.

———. "Logic and the Reification of Universals." In *From a Logical Point of View, 2nd Edition, Revised*, Cambridge: Harvard University Press, 1980b, 102–129.

———. "Reference and Modality." In *From a Logical Point of View, 2nd Edition, Revised*, Cambridge: Harvard University Press, 1980c, 139–159.

———. "Reply to Charles Parsons." In *The Philosophy of W. V. Quine*, edited by Hahn, and Schlipp, La Salle: Open Court, 1986, 396–403.

Rea, Michael. "The Probelm of Material Constitution." *The Philosophical Review* 104 (1995): 525–552.

———. "Introduction." In *Material Constitution*, edited by Rea, Lanham: Rowman and Littlefield, 1997a, xv–lvii.

Rea, Michael, editor. *Material Constitution: A Reader*. Lanham: Rowman and Littlefield, 1997b.

Roper, Andrew. "Towards an Eliminative Reduction of Possible Worlds." *Philosophical Quarterly* 32 (1982): 45–59.

Rosen, Gideon. "Modal Fictionalism." *Mind* 99 (1990): 327–354.

———. "The Refutation of Nominalism (?)." *Philosophical Topics* 21 (1993): 149–186.

———. "Abstract Objects." In *The Stanford Encyclopedia of Philosophy*, edited by Edward N. Zalta, Fall 2001. URL http://plato.stanford.edu/archives/fall2001/entries/abstract-objects/.

Rowe, William. "Modal Versions of the Ontological Argument." In *Philosophy of Religion: An Anthology*, Belmont: Wadsworth Publishing Company, 1994, 74–78. Second edition.

Roy, Tony. "Worlds and Modality." *Philosophical Review* 102 (1993): 335–362.

———. "In Defense of Linguistic Ersatzism." *Philosophical Studies* 80 (1995): 217–242.

———. "Things and *De Re* Modality." *Noûs* 34 (2000): 56–84.

———. "*About What There Is: An Introduction to Contemporary Metaphysics*.", 2013a. URL http://philosophy.csusb.edu/~troy/int-met.htm.

———. "*Symbolic Logic: An Accessible Introduction to Serious Mathematical Logic*.", 2013b. URL http://philosophy.csusb.edu/~troy/int-ml.htm.

———. "Making Sense of Relevant Semantics.", 2013c. URL `http://philosophy.csusb.edu/~troy/msrs-paper.pdf`.

———. "Technical Results for *Properties, Possibilities and Ordinary Things*.", 2013d. URL `tbd`.

Russell, Bertrand. "The Philosophy of Logical Atomism." In *The Philosophy of Logical Atomism*, edited by D. Pears, Lasalle: Open Court, 1985.

Shapiro, Stewart. *Foundations Without Foundationalism: A Case for Second-order Logic*. Oxford: Oxford University Press, 1991.

Shoenfield, Joseph. *Mathematical Logic*. Reading, Massachussetts: Addison-Wesley Publishing Company, 1967.

Sidelle, Alan. *Necessity, Essence, and Individuation: A Defense of Conventionalism*. Ithaca: Cornell University Press, 1989.

Sider, Theodore. "The Ersatz Pluriverse." *Journal of Philosophy* 99 (2002): 279–315.

———. "Reductive Theories of Modality." In *The Oxford Handbook of Metaphysics*, edited by Loux, and Zimmerman, Oxford: Oxford University Press, 2003, 180–208.

———. *Writing the Book of the World*. Oxford: Oxford University Press, 2011.

Smullyan, Arthur. "Modality and Description." *The Journal of Symbolic Logic* 13 (1948): 31–37.

Thomasson, Amie. *Fiction and Metaphysics*. Cambridge: Cambridge University Press, 1999.

———. "Speaking of Fictional Characters." *Dialectica* 57 (2003): 205–223.

Unger, Peter. "The Problem of the Many." *Midwest Studies in Philosophy* 5 (1980): 411–67.

Van Inwagen, Peter. "Creatures of Fiction." *American Philosophical Quarterly* 14 (1977): 299–308. Reprint in van Inwagen, *Ontology, Identity and Modality*, 37-56.

———. "Fiction and Metaphysics." *Philosophy and Literature* 7 (1983): 67–77.

———. "Two Concepts of Possible Worlds." *Midwest Studies in Philosophy* 11 (1984): 185–213.

———. "Pretense and Paraphrase." In *The reasons of Art*, edited by Peter McCormic, Ottawa: University of Ottawa Press, 1985, 414–422.

———. *Material Beings*. Ithaca: Cornell University Press, 1990.

———. "Quantification and Fictional Discourse." In *Empty Names, Fiction and the Puzzles of Non-Existence*, edited by Anthony Everett, and Thomas Hofweber, Stanford: CSLI Publications, 2000.

———. *Ontology, Identity, and Modality: Essays in Metaphysics*. Cambridge: Cambridge University Press, 2001.

———. *Metaphysics*. Boulder: Westview Press, 2002, second edition.

———. "Existence, Ontological Commitment, and Fictional Entities." In *The Oxford Handbook of Metaphysics*, edited by Michael Loux, and Dean Zimmerman, New York: Oxford University Press, 2003, 131–157.

———. "A Theory of Properties." In *Oxford Studies in Metaphysics, vol I*, edited by Dean Zimmerman, New York: Oxford University Press, 2004, 107–138.

Walton, Kendall. *Mimesis as Make-Believe: On the Foundations of the Representational Arts*. Harverd University Press: Cambridge, MA, 1990.

———. "Metaphor and Prop Oriented Make-Believe." In *Fictionalism in Metaphysics*, edited by Mark Kalderon, Oxford: Oxford University Press, 2005, 65–87.

Weatherson, Brian. "Many Many Problems." *The Philosophical Quarterly* 53 (2003): 481–501.

Wiggins, David. "On Being in the Same Place at the Same Time." *The Philosophical Review* 77 (1968): 90–95. Reprint in Rea, *Material Constitution*, 3-9.

———. *Sameness and Substance*. Cambridge, MA: Harvard University Press, 1980.

Williamson, Timothy. *Modal Logic as Metaphysics*. Oxford: Oxford University Press, 2013.

Wilson, Robert. "Two Views of Realization." *Philosophical Studies* 104 (2001): 1–31.

Wolterstorff, Nicholas. *Worlds and Works of Art*. Oxford: Clarendon Press, 1980.

Wright, Crispin. *Frege's Conception of Numbers as Objects*. Aberdeen: Aberdeen University Press, 1983.

Yablo, Stephen. "Identity, Essence, and Indiscernibility." *The Journal of Philosophy* 84 (1987): 293–314.

———. "Does Ontology Rest on a Mistake?" *Proceedings of the Aristotelian Society* 72 (1998): 229–261.

———. "Go Figure: A Path Through Fictionnalism." *Midwest Studies in Philosophy* 25 (2001): 72–102.

Zalta, Edward. *Abstract Objects: An Introduction to Axiomatic Metaphysics*. Dordrecht: D. Reidel Publishing Company, 1983.

———. *Intensional Logic and The Metaphysics of Intentionality*. Cambridge, Massachusetts: The MIT Press, 1988.

———. "Natural Numbers and Natural Cardinals as Abstract Objects: A Partial Reconstruction of Frege's *Grundgesetze* in Object Theory." *Journal of Philosophical Logic* 28 (1999): 619–660.

Zimmerman, Dean. "Theories of Masses and Problems of Constitution." *Philosophical Review* 104 (1995): 53–110.

———. "Temporal Parts and Supervenient Causation: The Incompatibility of Two Humean Doctrines." *Australasian Journal of Philosophy* 76 (1998): 265–288.

———. "One Really Big Liquid Sphere: Reply to Lewis." *Australasian Journal of Philosophy* 77 (1999): 213–215.